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GENERAL PROBLEMS OF THERMODYNAMICS

Relativity and Temperature[1]

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Abstract—Phenomenological considerations demonstrate that temperature and pressure are invariant with Lorentz transformations. Consider two observers, one at rest and the other at relative speed $\beta = u/c$ with respect to a thermodynamic system. If the system is in equilibrium at a triple point of a given P–T diagram, both observers will see three phases in equilibrium, independent of β. For instance, these could be vapors, ice, and water. Therefore, both will conclude that the system is at a temperature of the triple point (273.16 K for H_2O). Temperature reflects the equilibrium structure of the system. If there is a temperature dependent distribution of cyclic and linear molecules in an equilibrium system it will be observed unchanged from all frames. Therefore, all observers will have to determine the same temperature independent of the relative speed of their frames. The important experimental indication is the distribution law of temperature of distant galaxies. Because of the expansion of the Universe they move at a relatively high speed. Therefore, according to Planck's transformation law of temperature they *should be cold and invisible*. On the other hand, according to Ott *they should be infinitely hot and bright*. As soon as their temperature is within the "normal" limits one has to conclude that the temperature is invariant with Lorentz transformations. If temperature is invariant with speed, then entropy with respect to the Boltzmann constant is not. This put serious problems on the statistical physics.

INTRODUCTION

Relativistic generalization of thermodynamics was considered by the founders of relativity and quantum mechanics [1–5]. In 1907, Planck assumed that the first and the second laws of thermodynamics keep their form in all inertial frames. He found that if Q_0 is the heat of a system at rest with its inertial frame K_0, then a distinct inertial reference frame K_u, moving with uniform velocity u with respect to K_0, would ascribe to this system a lower heat content

$$Q_u = Q_0(1-\beta^2)^{1/2}, \tag{1}$$

where $\beta = u/c$. Hereafter, all variables with subscript 0 refer to the inertial frame at rest with the observer, while variables with subscript u refer to inertial frame K_u moving at relative speed β. Planck managed to demonstrate that pressure is invariant with the Lorentz transformations

$$P_u = P_0. \tag{2}$$

Assuming invariance of the entropy, Planck concludes that temperature T_u should transform in the same way as the heat does

$$T_u = T_0(1-\beta^2)^{1/2}. \tag{3}$$

However, in 1963, this result was challenged by Ott [6] who reached exactly the opposite transformation law

$$Q_u = Q_0/(1-\beta^2)^{1/2}. \tag{4}$$

[1]This article was submitted by the author in English.

The difference originates from the different definitions of forces and impulses. Applying the same assumption of invariance of entropy, Ott's approach leads to

$$T_u = T_0/(1-\beta^2)^{1/2}. \tag{5}$$

Later on, the results of Ott were supported by Arzelies [7]. Recently, Landsberg and Matsas [8], considering the Unruh–DeWitt detector, concluded that continuous Lorentz transformations of temperature cannot exist for black body radiation.

According to van Kampen [9], it is impossible to measure the temperature of a fast moving system because it is impossible to establish good thermal contact. The argument is rather weak. It is just as difficult to establish good thermal contact with a fast moving object as it is to attach to it an (immobile with K_0) etalon of length in order to determine its size. Meanwhile, the length transformation law is well known and plays an important role in relativity. The temperature plays a key role for thermodynamics. Therefore, to declare temperature immeasurable is equivalent to declaring thermodynamics not existing in relativity theory. *Unlike the situation with the Heisenberg uncertainty principle, one should not formulate a principle of immeasurable temperature.* Temperature can be measured from a distance and without establishing thermal contact with the object. For instance, it is sufficient to have a good pyrometer. An illustration of one possibility of measuring the temperature of fast moving objects is given by Landsberg and Matsas [8]. This is the first result indicating that temperature is invariant with the speed of the inertial frame.

Consider we observe from K_0 two systems at rest exchanging heat with frame K_u, we can certainly determine which of them has the higher temperature. Moreover, we can decide, by investigating the heat exchange, between a system at rest with K_0 and another one at rest with K which of them has the higher temperature.

Here we stay on the Planck viewpoint that temperature does exist, it can be measured and, therefore, the relativistic approach to thermodynamics is possible. According to the first Einstein postulate, the laws of physics are the same in all inertial reference frames. Note that no thermal equilibrium nor thermal contact needs to be established between the observer and the investigated system. Therefore, no interchange of energy (heat), or of momentum between them is needed.

The attempt to declare temperature immeasurable in relativity theory originates from the intuitive wish of some scientists to avoid the problem that one can determine, apparently, different Lorentz transformations for temperature depending on the theoretical approach and/or on the experiment considered. The problem originates from a wrong understanding of the physical meaning of temperature and, what is more important, from the use of some basic constants of classical theory in the case of relativity without considering their possible changes.

According to the second basic postulate of thermodynamics, every equilibrium system is completely determined by the set of external variables (volume, pressure, magnetic field, etc.) plus one internal variable named temperature. Most of the systems, in particular melts, have equilibrium structures sensitive to temperature. Therefore, temperature is *first* a measure of the structure of the equilibrium system. And *second*, it is very popular to define temperature as a measure of the energy of internal motion of molecules. These two definitions lead to the apparent dualism in the relativity treatment of temperature. The structure is rather a quality (for instance this could be the temperature dependent distribution of cyclic and linear molecules in $LiPO_3$ melt, or a distribution of the number of monomers creating large molecules in some complex systems) and therefore it is invariant with speed. Note that, in the examples given here, all observers in all frames, no matter inertial or not, will see the same distributions and it must be concluded that the system is at the same temperature.

On the other hand the energy, a well-defined quantity, is not invariant with speed. The explanation is that not the temperature but its product with Boltzmann constant k_B gives the temperature. Therefore, one has to assume that the Boltzmann constant is not invariant with speed while temperature, reflecting the internal equilibrium structure of the system, is invariant.

For instance, if the system consists of three equilibrium phases (triple point), the temperature is known. The data of any thermometer should be calibrated according to this temperature. Say the triple point of water is by definition at 273.16 K and all observers from all frames, no matter fast moving or not, inertial or not, will see the three phases in equilibrium. This means that they will decide the system is at the same temperature $T_u = T_0$.

The light spectrum of the moving system depends on the product of Planck constant h, of Boltzmann constant k_B, and of temperature T. *If the Planck transformation (Eq. (3)) is valid, one must conclude that fast moving galaxies should be cold and invisible. On the other hand, with the Ott's transformation (Eq. (5)), these stars should be infinitively bright.* As soon as either of these happens, one has to expect that temperature is invariant with the speed.

PHENOMENOLOGICAL CONSIDERATIONS

An indication that the Lorentz transformation for the product of the Boltzmann constant and temperature is similar to the one proposed by Planck (similar to Eq. (3)) originates from the equation of the state of ideal gas

$$PV = nk_BT. \tag{6}$$

Note that Planck (see [1–3]) has demonstrated that pressure P is invariant with the speed while volume has the transformation of length. Since both sides of Eq. (6) should follow the same transformation law, it follows that

$$(k_BT)_u = (k_BT)_0(1-\beta^2)^{1/2}. \tag{7}$$

It should be noted that Eq. (7) is in agreement with Planck's transformation law for the element of heat, Eq. (1). Planck has assumed that Boltzmann constant k_B is invariant, so that the temperature follows Eq. (3). We have demonstrated that temperature is invariant

$$T_u = T_0 \tag{8}$$

and therefore

$$(k_B)_u = (k_B)_0(1-\beta^2)^{1/2}. \tag{9}$$

It should be noted that the above result can be obtained also from the entropy expression. Since temperature is invariant, with Eq. (1) it follows that

$$S_u = S_0(1-\beta^2)^{1/2}. \tag{10}$$

The simplest explanation of Eq. (10) is the assumption that the Boltzmann constant is not invariant with speed. To confirm this difficult statement, we give several examples.

(1) Consider a system in which two processes takes place with rates depending on temperature in a different manner. The ratio between the two rates will be the same in all frames only if temperature is invariant with the relative speed. Note that this is a requirement not only for equilibrium systems (two rates are equal and

the second process is the reverse of the first one) but for stationary systems also and even for far from equilibrium systems in which (at constant temperature) the two rates change with time. In the last case, the time change for the observer at rest with K_u should be corrected with the transformation for the time–space.

(2) Life can exist in a relatively narrow temperature interval. Every observer, no matter how far away nor how fast it is moving, that sees our civilization has to conclude that the temperature of the Earth is within a narrow region.

(3) Consider an **equilibrium thermodynamic system** at rest with respect to the frame K_0. If the system is at a triple point of the corresponding *P–T* diagram, three phases will coexist. For instance, these could be graphite, diamond, and liquid carbon (at extremely high pressure and temperature) or it could be the triple point of ice and water, etc. The same three phases will be seen from any other frame K. As soon as they can coexist at one P, T point only, the temperature and the pressure of this system will be the same in all frames. Temperature is measured by a thermometer calibrated at certain equilibrium points. The one best recommended is the ideal gas bulb thermometer calibrated at 0 K and at the triple point of water 273.16 K. If an observer at rest with K_0 sees ice and water in equilibrium then all observers will see the same phases in equilibrium and will conclude that the system is at the same temperature. Therefore, one can conclude that *pressure and temperature are invariant with respect to the Lorentz transformation.*

(4) Temperature is invariant even for **nonequilibrium systems**. Let us consider, for instance, a system consisting of a charged battery connected to a heater with a link that is conducting at some temperature interval and isolating outside this interval. For example, it could be a wire of a high temperature superconductor being an isolator at room temperature. If the observer at K_0 is at room temperature he will see no electrical current and the battery will remain charged. At the same time, the observer at K should see (if temperature is not invariant) that the battery is discharging, heating the surrounding area. As this process is irreversible, the observer will never see the opposite process with the surroundings cooling spontaneously and recharging the battery even if it accelerates. Therefore, an absurd situation will appear if the second observer arrives to the first one at K_0. At the same space–time point, the first observer should see the battery charged, while the second one should see it uncharged. The conclusion is that *temperature is invariant with speed, for both equilibrium and nonequilibrium systems. It is invariant even if the framework of the observer outside the system is noninertial.* This does not mean that temperature is invariant if the system is accelerating or introduced into a strong gravity field. The latter will be a problem for future investigation.

Although the general assumption of Boltzmann relates the entropy and the statistics, the main result is that there is a constant relating temperature to the energy of internal motion of molecules. In classical theory this constant is universal. It is independent of the chemical composition, the mass of molecules, the nature of the phase, etc. In relativistic theory, one has to assume again that this constant remains independent of all possible variables just like in the classical case. *However, it turns out that it is not independent of the speed.*

The set of relativity transformation laws for the volume V, temperature T, and pressure P

$$\begin{cases} V = V_0(1-\beta^2)^{1/2} \\ P = P_0 \\ T = T_0 \end{cases} \tag{11}$$

is sufficient to determine the Lorentz transformation for other thermodynamic variables. Following the same logics, one can derive the transformation laws from the known thermodynamic relations.

Temperature is a scalar; therefore it cannot depend on the direction. Let us compare two thermometers. The bulb of the one that is $\perp$ to the motion axes will be unchanged, while the bulb of the thermometer coaxial with the direction of motion will be shorter. However, this does not mean that temperature is lower. This simply means that the thermometer has to be recalibrated because at every point there is only one temperature.

It is important to study the temperature distribution law of distant galaxies. Because of the expansion of the universe their speed is very high. According to Eq. (3), they *should be cold and not seen.* According to Eq. (5), however, *they should be infinitely bright*. Note that there is an important difference between the frequency shift of light caused by the Doppler effect and a shift caused by temperature change. According to Planck's formula, the intensity of light I depends on frequency ν and temperature T as follows:

$$I = \frac{c^3}{8\pi h}\frac{\nu^3}{\exp(h\nu/k_B T)-1}. \tag{12}$$

It is seen that it is possible to distinguish the intensity change caused by the Doppler effect from the shift caused by the Lorentz transformation of the $h/k_B T$ product. Moreover, the Doppler effect depends on whether the object is moving towards or away from the observer.

CONCLUSIONS

The temperature is invariant with respect to Lorentz transformations. This comes from the requirement of invariance of the triple points in *P–T.* At the same time, the product of entropy and temperature must have a relativity transformation equivalent to that predicted by Planck for the element of heat. This phenomenon is a tempting problem for the statistical thermodynamics. It seems the simplest explanation is that the Boltzmann constant is not invariant with respect to the relative speed of the object.

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