Spiral Galaxy NGC 7424
(VLT MELIPAL + VIMOS)

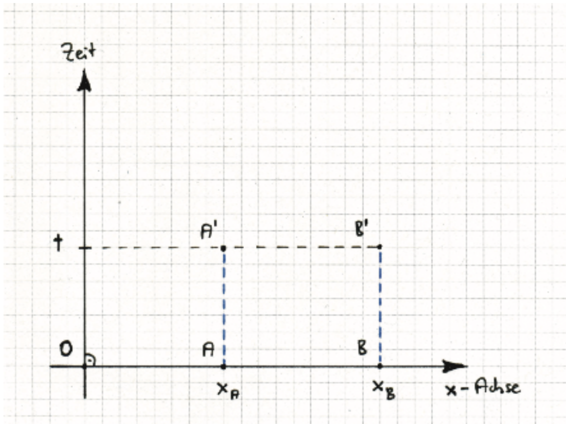
ESO PR Photo 33c/04 (1December 2004)

H Epstein Diagrams for the General Theory of Relativity

We learn from Epstein diagrams how objects spontaneously begin to fall, and how clocks tick more slowly under the influence of stronger gravitation. To do so we need only to bend our familiar Epstein STR diagrams a bit ... The nice thing is that we can maintain the basic axiom: Everything always moves through space-time at the speed of light. Out of this arises a new variant of the twin paradox. As usual we'll investigate this quantitatively. We then interpret the results using the principle of maximum proper time. The fifth section presents Epstein diagrams in an elegant rolled version. Finally, we let Epstein show us how the curvature of space alone bends the inertial trajectories of fast objects. Here we anticipate qualitatively some experiments that we will handle quantitatively in **I**.

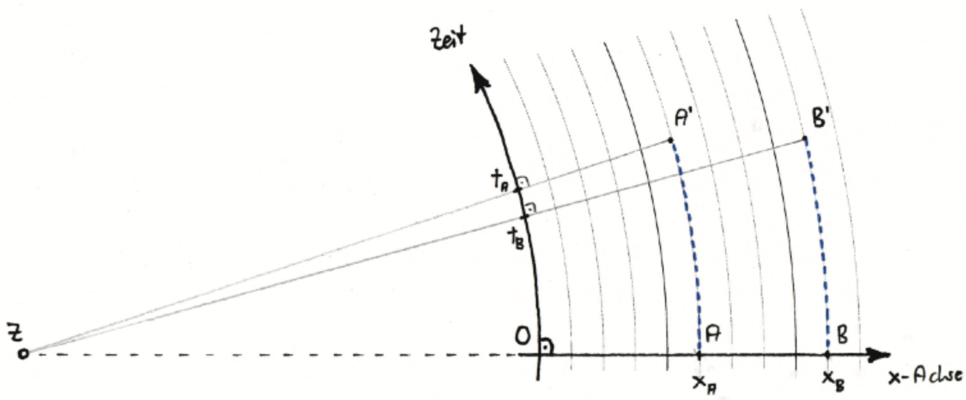
H1 Gravitation and the Curvature of Space-Time

Below is a space-time diagram, as we learned to draw with STR in section **C**:



Clocks are placed at the positions x_A and x_B and both have been set to zero in A and B. Through the projection of the space-time position of the clocks on the time-axis, we can determine what time the clocks show at any position. One can use the time-axis to read the elapsed proper time for each object since the start of an operation.

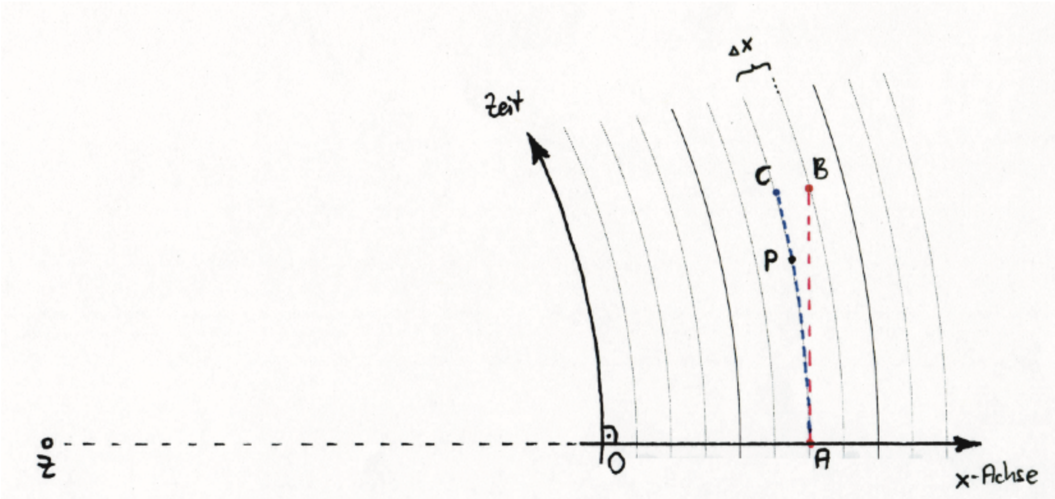
Let the gravity at position x_B be stronger than at x_A. What can be done so that along the path BB' time runs a bit less quickly than along the path AA', which is of *exactly the same length* in space-time? Epstein shows us the simple yet elegant solution: we only need to bend the time-axis a bit away from B to produce the desired effect!



Objects that linger at point x_A move on a circle centered at Z with radius ZO + x_A. The corresponding applies to stationary objects at x_B. It is important that the two arc segments AA' and BB' are exactly the same length. This arc length specifies how long the process lasted for an observer in the OFF. We can read the elapsed proper time for A or B, by projecting A' or B' vertically on the now circular time-axis. We need only link A' and B' with Z. The clock at B' apparently shows less time than the one at A'!

The curvature of the time-axis, i.e., the inverse of the radius ZA = ZA', is critical for the strength of the effect at point x_A. This curvature is a function of the distance r from the center of the central mass. This is to the right of x_B, because gravity is stronger at x_B than x_A. If the curvature is zero, i.e., the radius ZO is infinitely large as in the first Epstein diagram above, then there is no effect from gravity and we are back in the STR.

We have now described the behavior of static objects in a gravitational field. As examples of such objects, imagine an apple in the Netherlands or a pine cone in the Swiss Alps each of which is hanging fixed on its branch. But what happens when the connection between the stem and the fruit breaks? The Epstein diagram holds the answer to this question:



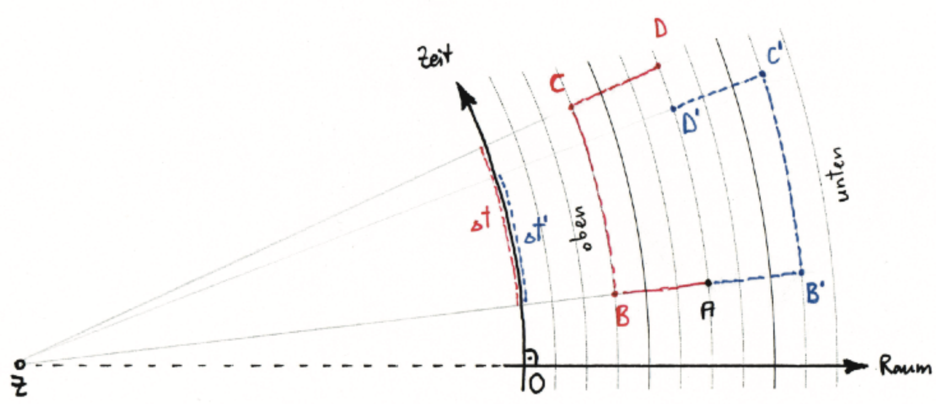
Apple and branch are located at time zero at point A in space-time. Just then the connection between the two is cut. The branch moves up as fixed from A to C, while the apple moves along the tangent to arc AC, i.e., along the red dotted line and arrives at B! For the freely falling apple, there is no longer gravity. For the apple it is rather as if the tree is being prevented by the ground from following an inertial trajectory. After a short time, a distance Δx opens between the apple and the branch, whereby the apple has moved in the direction of the center of mass. The arc length AC and the segment length AB are again exactly the same length.

The branch traverses the arc AC proportional to time. At point P on the arc, the angle between the tangent to the circle at P and the red line AB is equal to the angle PZO. This angle indicates, however, how quickly the apple is moving away from the branch at a given moment, that is, the relative velocity of the apple and branch. Thus we can even read Galileo's law for falling bodies from the diagram: Regardless of its mass the apple has a speed proportional to the elapsed time! Recall, however, that this law has a limited scope: v must be much smaller than c!

The angle between two curves in the space-time diagram still indicates a relative velocity. More precisely: $\sin(\phi) = v/c$! The maximum angle is still 90°. Light moves, as usual, perpendicular to the time-axis. However, in the case of apples falling at the earth's surface, a quantitative analysis of the intermediate angle is tricky, as section **H3** will show. But before doing so we would like to use our bent space-time diagrams to present a new variant of the twin paradox.

H2 Twins Again, With Different Rates of Aging

A picture is worth a thousand words:



Epstein’s twins, Peter and Danny, decide they no longer want to be the same age. Since time 'deep' in a gravitational field runs more slowly than 'higher up' in the field they decide while having tea that Peter will spend some time on the second floor while Danny hangs out in the cellar. At point A, they begin to implement this plan. The lines AB and AB' and CD and C'D' are of equal length. But also the arcs BC and B'C' are of equal length, so the two will meet again at the same time on the first floor, i.e., at the place of A, D' and D. Peter is now really a bit older than Danny!

What does it mean, however, ‘to meet again at the same time’? We assume again the position of an observer in OFF. Thus, for us the whole experiment elapses in time AB + BC + CD = AB' + B'C' + C'D'. For the red (Peter) the time elapsed is given by the projection of the arc BC on the time axis (i.e., the red Δt), while for the blue (Danny) even less time has elapsed. For him it is given by the projection of B'C' on the time axis (i.e., the blue $\Delta t'$).

Did you notice that Peter and Danny were very quick at climbing stairs? How fast were they moving? Well, the whole story would also work if they took it somewhat more relaxed. The drawing would, however, be a bit more complicated.

The Swiss lawyer, philosopher and chansonnier Mani Matter has written a song about a boy running 'as fast as lightning':

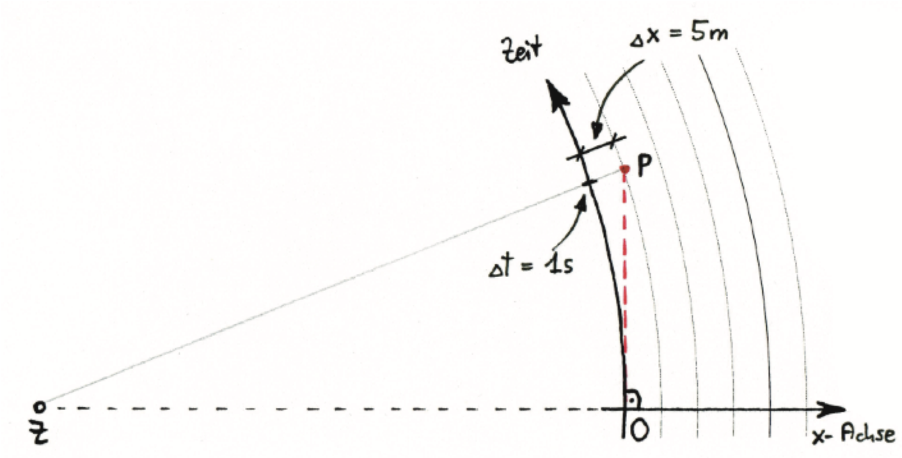
> there is a boy let's call him Fritz, there is a boy let's call him Fritz
> that can run as fast as a blitz, that can run as fast as a blitz.
>
> he runs that outrageous athlete, he runs that outrageous athlete
> so fast you can not see his feet, so fast you can not see his feet.
>
> and 'cause he never ends his race, and 'cause he never ends his race
> no one has ever seen his face, no one has ever seen his face.
>
> so even I, the bard, agree, so even I, the bard, agree
> perhaps the lad can't even be, perhaps the lad can't even be.
>
> Lyrics and tune by Mani Matter [46-41], translated from the original Schwiizertüütsch (Swiss German) by David Eckstein and Samuel Edelstein

Fritz becomes a victim of Ockham’s razor! (see problem 1)

H3 A Quantitative Consideration

How far does an apple fall in one second from initial rest? Assuming for g the rounded value of 10, we get from $\Delta h = 0.5 \cdot g \cdot \Delta t^2$ a distance of 5 m. But one second of time on our scale diagram corresponds to a length of 300,000,000 m! If a second on the diagram has a length of 3 cm, a distance of 5 m in the diagram has a length 0.5 nm (nanometers) - which corresponds to just a few atomic diameters!

Let's calculate the length of the radius of the time axis (or, the inverse value, its curvature) when the space-time diagram is adjusted to the strength of the gravitational field at the Earth's surface:



We express all lengths in our space-time diagram in meters. The leg OP of the right triangle ZOP has, to a good approximation, the length $c \cdot 1s = 3 \cdot 10^8$ m (the final result will show that this approximation is actually extremely precise ...). The Pythagorean Theorem now gives us:

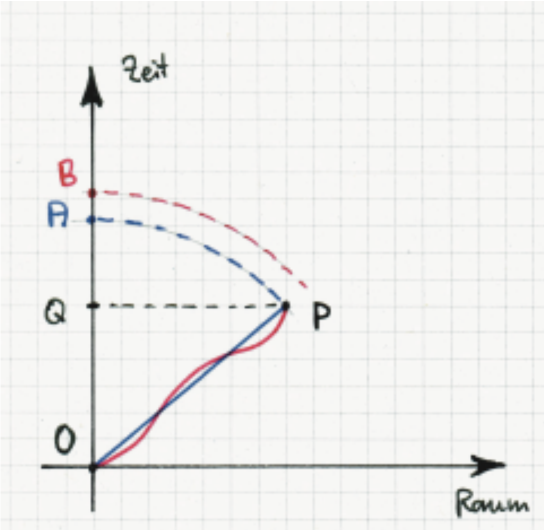
$$ZP^2 = ZO^2 + OP^2 \quad ; \quad (ZO + 5)^2 \approx ZO^2 + (3 \cdot 10^8)^2 \quad ; \quad ZO^2 + 2 \cdot ZO \cdot 5 + 5^2 \approx ZO^2 + 9 \cdot 10^{16}$$

$$10 \cdot ZO \approx 9 \cdot 10^{16} - 25 \quad ; \quad ZO \approx 9 \cdot 10^{15} - 2.5 \quad \text{and thus} \quad ZO \approx 9 \cdot 10^{15}$$

If the curvature of the time axis fits the strength of gravity on earth's surface, then the radius ZO must be $9 \cdot 10^{15}$ meters, which is almost exactly one light year! For comparison: the nearest fixed star is about 4 light years away from us and the semi-major axis of Pluto's orbit is only $5.9 \cdot 10^{12}$ m. The correct radius is therefore immense, and the correct curvature is minuscule. We are talking about a really weak gravitational field – whose curvature cannot be detected by the naked eye. But it is this tiny space-time curvature on earth's surface that causes freely moving objects to fall and that makes climbing stairs so strenuous!

Adam Trepczynski has created a Shockwave animation, which allows one to play with Epstein space-time diagrams - with and without gravity:
http://www.relativity.li/uploads/flash/epstein_space_time.swf

H4 Principle of Maximum Proper Time



No matter which path a clock travels from O to P through space-time - the time elapsed on the clock will always be OQ as measured by any inertial observer. The red and blue path, however, differ in that our 'black' inertial observer measures different elapsed times on his own clocks for the blue and the red path. For the blue path, it is simply the length of the segment OP = OA. But since everything always travels the same distance through space-time, we need only stretch the red path to find the corresponding elapsed time OB for black.

The proper time OQ that elapses on the clock is thus independent of the chosen path, but not, however, the quotient formed from this proper time and the time elapsed for an observer from any other inertial frame. For the straight blue path from O to P the qotient OQ / OP = OQ / OA has its maximum value, compared to all other paths from O to P as seen from the arbitrary chosen black system. The quotient is maximal because the denominator OA is minimal!

The name "principle of maximum proper time" is a bit misleading in this context since what is actually maximized for the blue path is the ratio $\Delta\tau$ / Δt, when we, like Epstein, denote proper time with τ and coordinate time with t. $\Delta\tau$ is measured on a single clock moving from O to P and influenced by gravitation and velocity, while Δt is measured on a clock of the distant observer in OFF, who in GTR really has a special position. While in STR we have given preference to symmetric presentations, we are now forced by gravitation to use the point of view of the distant observer in the OFF to compare other measurements.

Principle of maximum proper time: If no force is acting on a body, it moves from X to Y through space-time along a path for which the ratio $\Delta\tau$ / Δt is maximized.

In the STR, these paths are straight lines. Any other path through space-time is longer and thus increases the denominator Δt. Before we look at an example of the behavior in GTR, I would like to emphasize that here we do *not* consider the path that light takes from one location A to another location B through space - we will later. Here we consider "free fall lines" through space-time.

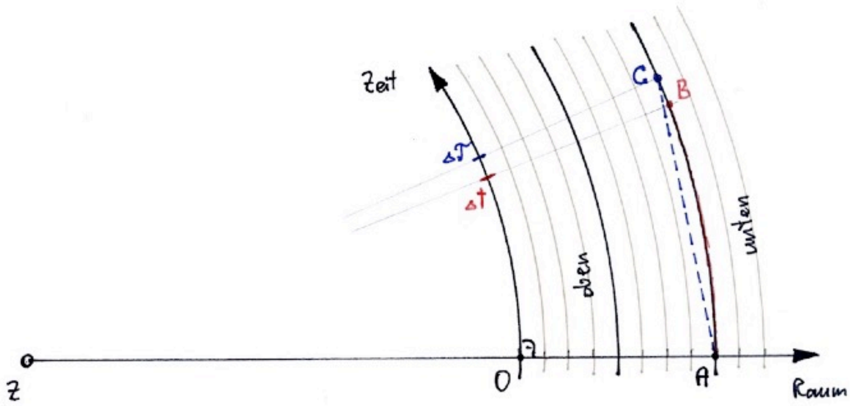
Now for an example:

What is the force-free path from a point A on the surface of the earth which leads 2 seconds later to the same place? It is a vertical throw with an initial speed of 10 m/s. During the first second the object rises, ever more slowly, to a height of 5 m and in the second second it falls freely from its height back to the starting point. We know the equations of motion and the path in space from Newtonian physics:

$h(t) = v_0 \cdot t - 0.5 \cdot g \cdot t^2 \approx 10 \cdot t - 5 \cdot t^2$ und $v(t) = v_0 - g \cdot t \approx 10 - 10 \cdot t$

Why does *this* movement satisfy the principle of maximum proper time, why is it not more advantageous simply to remain on the ground or to rise to a height of 20 meters?

First consider the related Epstein diagram:



According to Epstein's dogma, let the straight blue path from A to C have the same length as the red arc from A to B on the circle. The proper time interval Δτ belonging to the force free flight from A to C is obiously longer than the time intervall Δt measured with a clock at rest, moving through spacetime from A to B on an isotrpic arc.

Why is the blue path more advantageous? Because higher up the clock is running faster! Why does it not then go even further? Because then it must go up and down so fast that the time dilation due to the high velocity would negate the advantage of the greater height! GTR advocates height, STR advocates small speeds - and the optimal compromise is our vertical throw!

$$\frac{\Delta t(h) - \Delta t_0}{\Delta t_0} \approx \frac{g \cdot \Delta h}{c^2}$$

What time increase does the vertical throw bring? We already deduced in **G4** how the time difference depends on the height difference:

$$\int_0^2 \frac{g \cdot \Delta h}{c^2} \cdot dt = \frac{g}{c^2} \cdot \int_0^2 \left(10 \cdot t - 5 \cdot t^2\right) \cdot dt = \ldots = \frac{g}{c^2} \cdot \frac{20}{3} \approx \frac{200}{3 \cdot c^2}$$

We must sum the increase per unit of time for the entire flight time of 2 seconds. The corresponding integral is harmless

This is the *additional* elapsed proper time in seconds due to the "hop". We must balance this with the loss of proper time due to the velocity. According to the STR

$$\frac{\Delta t_0 \cdot \sqrt{1 - \frac{v^2}{c^2}} - \Delta t_0}{\Delta t_0} = \sqrt{1 - \frac{v^2}{c^2}} - 1 = \left(1 - \frac{v^2}{2 \cdot c^2} - \frac{v^4}{8 \cdot c^4} - \ldots\right) - 1 \approx -\frac{v^2}{2 \cdot c^2}$$

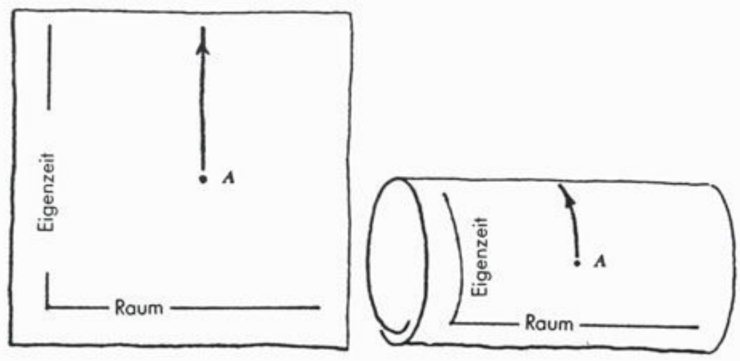
We know that $v(t) \approx 10 - 10 \cdot t$ and can therefore again integrate from 0 to 2 over time t. This integral is also quite harmless:

$$\int_0^2 -\frac{v^2}{2 \cdot c^2} \cdot dt = \frac{-1}{2 \cdot c^2} \cdot \int_0^2 (10 - 10 \cdot t)^2 \cdot dt = \frac{-100}{2 \cdot c^2} \cdot \int_0^2 (1 - t)^2 \cdot dt = \ldots = \frac{-100}{2 \cdot c^2} \cdot \frac{2}{3} = \frac{-100}{3 \cdot c^2}$$

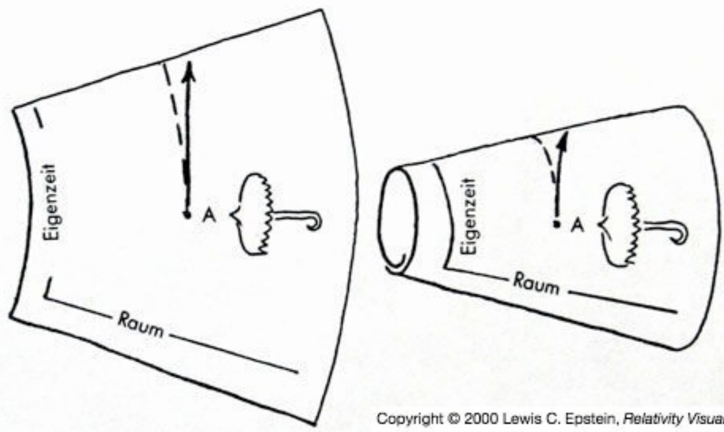
The loss from STR is thus only half as large as the gain from GTR and the overall positive balance is $+100 / (3 \cdot c^2)$ seconds. In problem 4, you will be asked to show that the increase of proper time is greatest for *this* parabola; that for higher paths the loss due to the STR increases faster than the gain due to GTR - and vice versa. The solution of Galileo and Newton corresponds exactly to the parabola which fulfills the principle of maximal proper time!

H5 Epstein Diagrams - Flat or Rolled

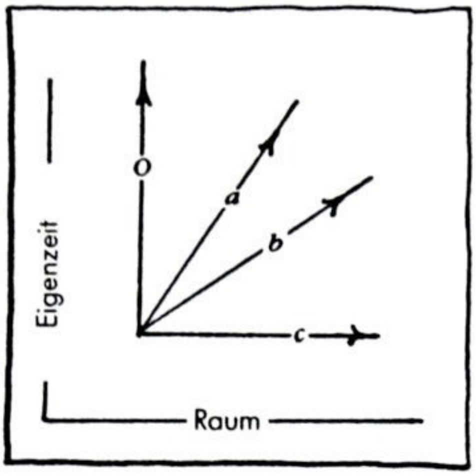
For periodic processes such as the up and down oscillations of a spring pendulum in a gravitational field or the somewhat fictitious free fall through a tunnel which traverses the center of the earth, the moving object repeatedly returns to the same place. Such processes can be very beautifully depicted in a rolled version of an Epstein diagram. The first picture shows a stationary object in both a flat and in a rolled Epstein diagram. The time axis is not curved; there is no gravity at work; we are in the realm of STR:



What do we get when we roll the curved diagram of section **H1**? A lampshade, or more scholarly, a truncated cone:

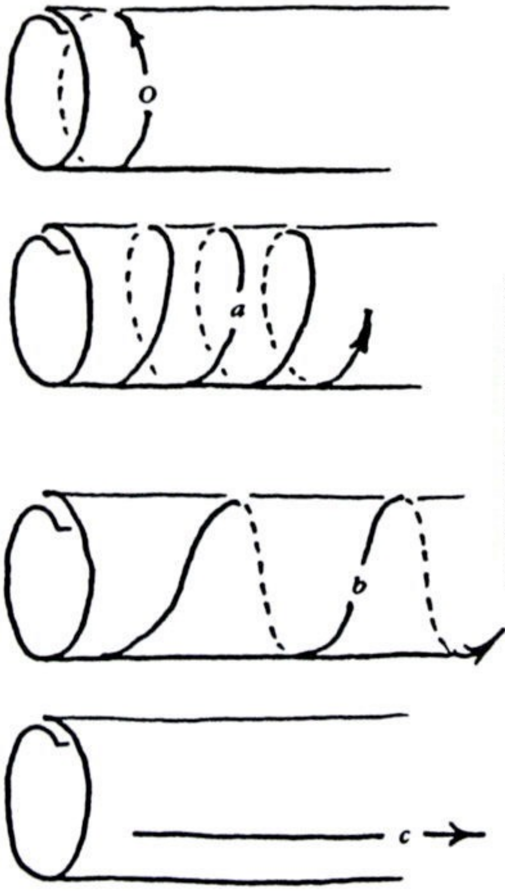


In his diagrams [15-146ff] Epstein occasionally suggests by using glasses or parasols the 'down' direction of the gravitational field (all diagrams in this section are taken from Chapter 10 of his book). We once again nicely see how an object that is not held in place by force moves in a straight line through space-time toward the 'bottom'. The angle ϕ between this 'fall-line' and the circles on the surface of the cone, which are at a fixed location and centered on the axis of symmetry of the cone, continually increases. $\sin(\phi) = v/c$ is still correct. It just indicates that the relative speed between fixed points and the free-falling object is increasing - just as we have found in section **H1**.



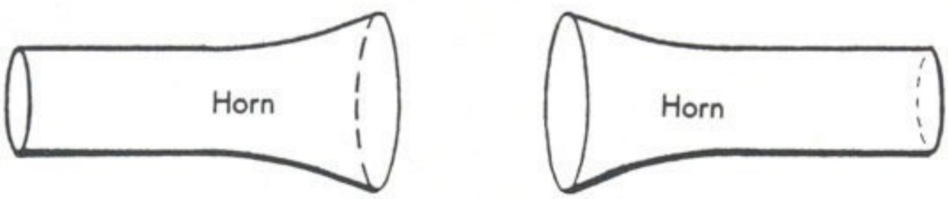
Let's consider the paths of various fast moving objects in gravity-free space-time. We are familiar with the representation in a plane: o lies at the origin, a is pretty fast, b is even faster, and c gives the behaviour of a photon.

To the right, all four speeds are represented in the rolled version. We get four simple paths on a cylindrical surface.

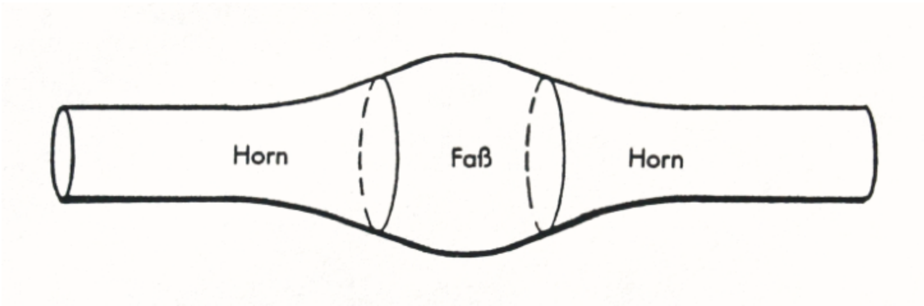


We now could study how this is presented in a homogeneous gravitational field, with g and thus the curvature of space-time being constant. A lampshade is a good representation of the *local* situation at a given distance from the center of the field generating mass. But large spatial movements can, however, not be represented!

Which is the corresponding solid of revolution, if the curvature is increasing with proximity to the Earth's surface - or, in other words, when an stationary orbit continually claims more proper time? The answer: Something that looks like the bell of a trombone or an ear trumpet. Epstein calls this body a horn.

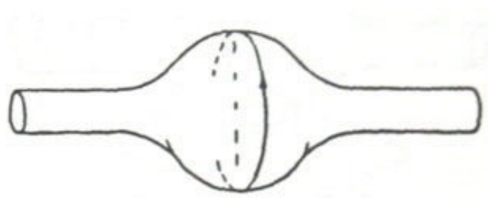


This diagram presents a large section of the x-axis on both sides of the earth together with the "rolled" time axis. The situation in the earth's interior must be presented in the gap in-between. There, however, gravity (and hence the curvature) decreases linearly toward the earth's center. Therefore, between the two horns a sphere with detached polar caps must be added.



The ‘barrel’ in the middle is actually an excised sphere, as we will presently prove. Imagine a cylindrical tunnel directly through the earth’s center from Switzerland to, let’s say, New Zealand. The tunnel axis is identical to our x-axis and the origin corresponds to the center of the earth. We can now, starting from any point on the x-axis, send an object, with or without an initial velocity, on a journey through the earth along the x-axis.

Objects falling near the earth swing like a spring pendulum back and forth around the center of the earth. In the interior of the earth the gravitational force obeys Hook’s Law: $F = -k \cdot \Delta x$. The oscillation period - as with the spring pendulum – is independent of the initial degree of deflection from the resting position, and thus the paths rolled into our space-time diagram must be the same length, regardless of the initial velocity! This is exactly satisfied if the ‘barrel’ between the two horns is a section of a sphere! The ‘straight’ paths, which correspond to free-falling are only on a sphere’s surface always closed, and also have in all cases the same length per orbit:



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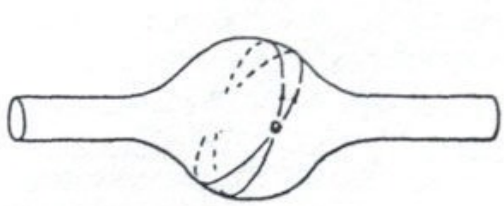

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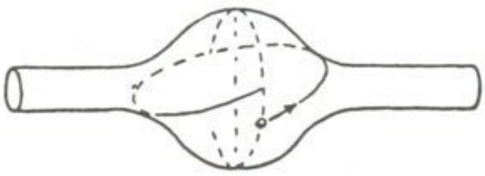
But throw an object with a positive initial speed in this tunnel, it will go beyond the spherical region and will also take somewhat more proper time per orbit than the pendulum inside the earth (sketch below on the left). If the initial velocity is even larger than the escape velocity of about 11.2 km/s, the object will escape (sketch below on the right):



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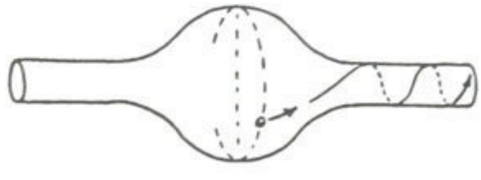

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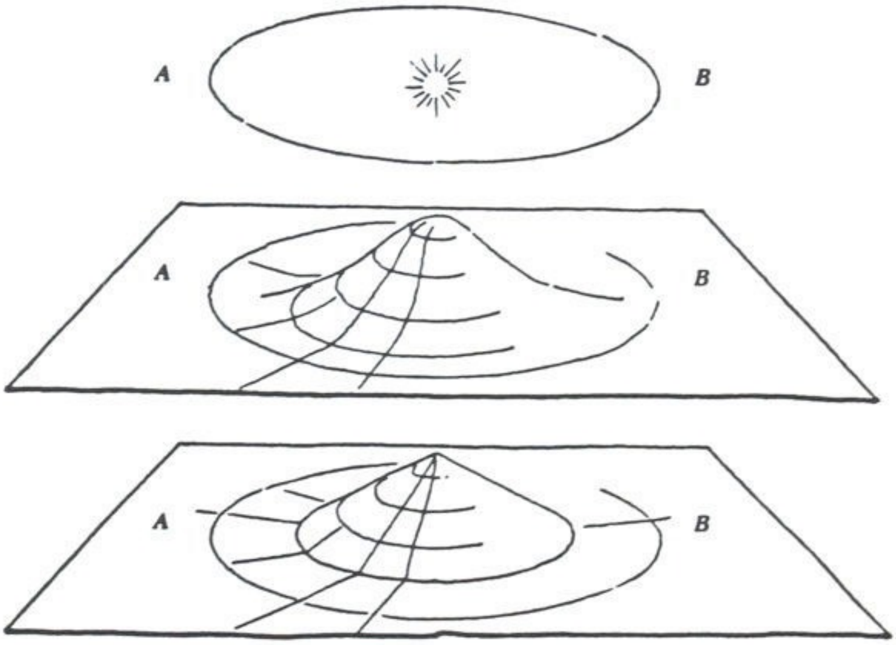
So much for the curvature of space-time as the cause of effects which we normally blame on forces in Newtonian physics. But that is only half the story: Even the metric of space itself is distorted. We will study these effects - again with Epstein - in the next section.

Adam Trepczynski has also produced a Shockwave animation of the rolled space-time diagrams of Epstein which he has kindly made available: http://www.relativity.li/uploads/flash/gravitation.swf

H6 Gravitation and the Curvature of Space

We have already seen in **G4** that the metric of space in the vicinity of a gravitational mass no longer obeys the laws of Euclid. For example, local yardsticks measure the diameter of the earth as slightly larger than its circumference divided by π. The effect in a weak gravitational field such as that of the earth is again very small, but in the vicinity of the sun it is nowadays easily experimentally demonstrated.

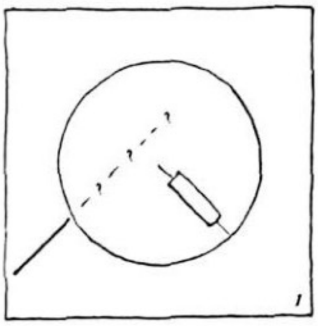
Consider again the diagram in **G4**. The dimple, or as Epstein says, the bump can help uncover the behavior of these metrics in an additional dimension that is not related to the z-direction. Epstein gives [15-165ff] detailed instructions on how you yourself can build such a model of a plane through the center of the sun (i.e., the ecliptic). The continuous changes in curvature are ignored for simplicity's sake and the whole bump is represented by a cone:



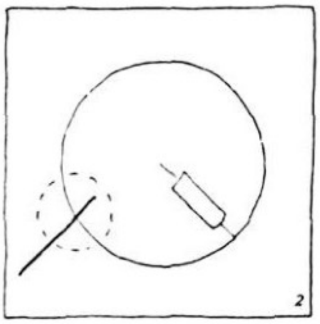
This simple model shows qualitatively all of the effects which according to the GTR have their source in the non-Euclidean metric of space!

I trust the reader can craft such a cone themselves with no additional instructions. Keep in mind that it must be possible to 'open' the cone and spread it flat on a table and then afterwards again reform it into a cone. Therefore it is preferable to use the milky variety of scotch tape rather than the clear ...

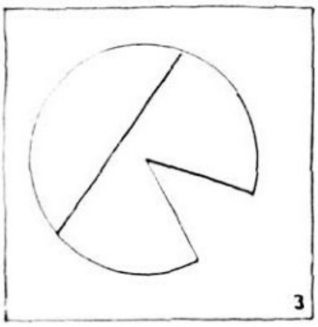
With this model we are investigating – following always [15-165ff] Epstein - what effect such a space bump has on the path of a light beam (further applications will follow in the next section I). It is truly awesome how Epstein using such simple means, accurately and clearly shows the effects of space curvature!



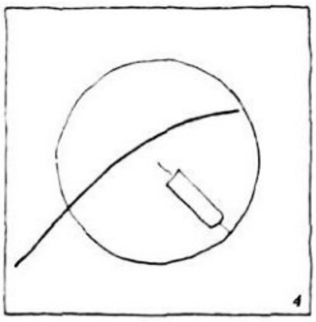
A beam of light approaches our space bump. Which path will it follow, when it enters the field of the non-Euclidean metric?



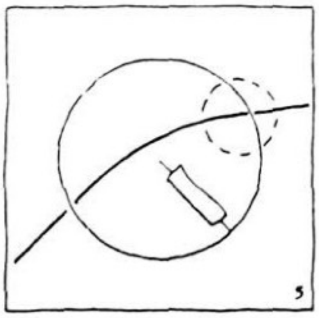
The transition from the diagram's flat surface to the curved cone is elegant: Remember that the bump rises softly from the flat surface and that the edge does not really exist! Flatten the cone locally just a bit (dotted circle) and extend the linear beam inside the dashed circle.



The beam will continue to spread out in a 'straight' line. But what does this mean on the surface of a cone? You can easily answer this if you uncoil the cone onto the diagram's flat surface. Extend the small, straight piece you already have inside the dashed circle of the cone mantle until the beam of light again leaves the cone.



Return the cone to its 3D shape. Put it in exactly the same position as it already had in Figure 2. Thus we have – as viewed from above - the conditions of a distorted geometry of space produced near a large mass. We will now see which path the light beam follows around the center of this mass.



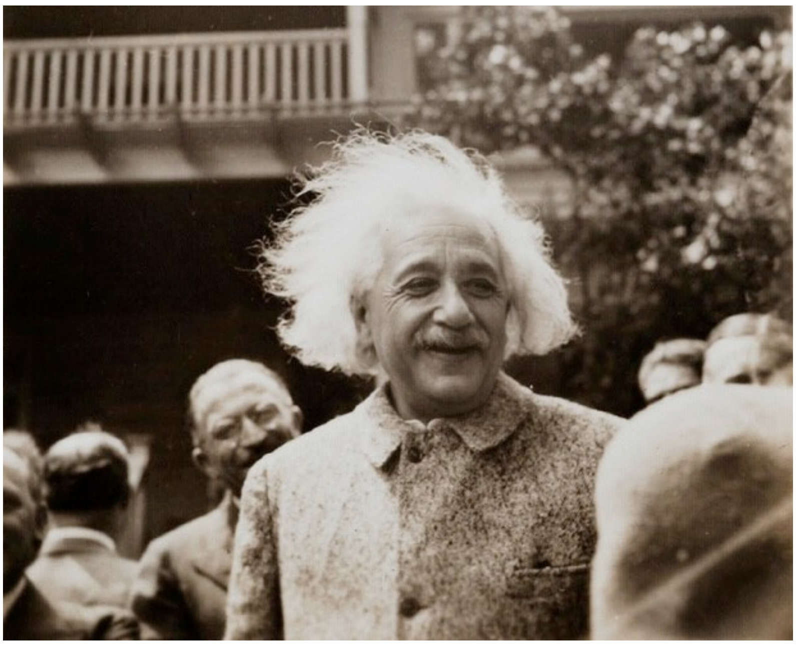
Exactly as in step 2 we now construct a further path. Press the cone in the dotted circle flat, and extend the direction of the path at the cones edge into the plane - done!

Light particles in a gravitational field fall (like everything else) in the direction of the mass center due to the curvature of space-time. Since this is independent of the mass of the falling particle, one was able to calculate this effect even before one knew the inertial mass of a light particle. The 27-year-old Johan Soldner submitted in 1803 an essay based on Newton's theory in which he calculated the deflection of a light beam passing the edge of the sun. The result of his calculation was an angle of 0.875 arc sec. This is precisely the value Einstein obtained in 1911 when he tested his newly conceived theory. In 1916 the 'completed' GTR provided a prediction of 1.75 arc sec, i.e., twice the value. This angle was now large enough that the possibility existed to clearly measure it from a photograph of stars in the sun's vicinity during a solar eclipse. This was achieved in 1919 by two excursions of the 'Royal Astronomers' under the direction of Arthur Stanley Eddington.

It was space curvature that was still lacking in the theory of 1911. Both curvatures (space-time and space-space) contribute almost the same and double the total effect. So a difference to the predictions of Newton's theory arose and the result of the measurement was in favor of the GTR and against Newton. Two theories may be structured differently – but if they make the same predictions for all phenomena, it cannot be decided through an experiment which is preferable. Fundamentally an experiment can only *disprove* a theory - never prove it.

There is however a remarkable difference between the curvature of space-time and that of space: The spatial curvature only affects objects that are moving through space, it has no influence on objects at rest! In particular, the curvature of space cannot cause an object to begin falling. But if it falls then it affects its trajectory. That the apple begins to fall is attributable to the curvature of space-time alone.

Epstein offers an interesting analogy [15-174, footnote]: It is like the effect of electric and magnetic fields on charged particles. The electrostatic force is there and works, regardless of the speed of the particle. The magnetic field has no effect on a charge at rest; the Lorentz force is proportional to velocity. Here, unfortunately, the analogy stops: The effect of spatial curvature on the path of falling objects does *not* depend on their velocity, it is important only that they have a path through the area to follow. However, velocity has a big impact on how long the curvature of space-time has an effect. Here the analogy fits again, since this holds for a charged particle in an electric field as well.



Einstein, about 1945

H7 Problems and Suggestions

1. Read the Wikipedia entry on what is meant by “Occam's razor” or “Ockham's razor”. Then think about the ether, parallel universes and the extra 6-8 dimensions that modern string theories postulate.

2. Calculate as in **H3** the radius of curvature of a space-time diagram which depicts the situation on the surface of the sun.

3. Draw flat, unrolled Epstein diagrams for the 4 paths (o, a, b, c) of section **H5** as seen by an observer at rest at the origin.

4. Calculate the net gain of proper time according to GTR and the corresponding loss according to STR as described in **H4**, for a general parabola $v(t) = k - k \cdot t$ and $h(t) = k \cdot t - 0.5 \cdot k \cdot t^2$ describing an object that returns after 2 seconds to its place of departure. Take the first derivative to the total gain and show that for $k = g$ it is a maximum!

5. Bertrand Russell used for the "principle of maximum proper time" the somewhat rakish expression "principle of cosmic laziness". To what extent is that justified?

6. Obtain somehow a sphere attached to two “horns” as described in section **H5** (if necessary, simply use two funnels and two tubes). You can then use long, narrow strips of paper attached to this surface to determine the space-time paths of different objects that fall through the earth. What does the total length of the paper represent?

7. Let a light beam pass through the tunnel of the model of task 6. Do you see clearly that it takes a little longer than an observer in the OFF would expect? This is precisely the Shapiro effect, which we will calculate in **I3**.

8. Now shoot a rocket through the earth of the model of task 6. In contrast to the light beam the traversal time for the rocket is reduced thanks to gravity!

9. A sense of 'straight lines' on curved surfaces (so-called ‘geodesics’) can be obtained by wrapping an ace bandage around your ankle. Which path does the fabric ‘itself’ wish to take? (suggested by Hans Walser)

10. Read the first tree chapters of Kip S. Thorne's book “Black Holes & Time Warps - Einstein's Outrageous Legacy“ [34]

11. The principle of Fermat for the path of a light beam is valid even in GTR! What does this mean for a beam of light, that passes close to the edge of the sun?

12. Enjoy the two books by Harald Fritzsch in which Newton, Einstein and a Bernese physicist of the present discuss the STR and GTR ([35] for the STR and [36] for the GTR). Newton shows his brilliant and critical mind as he is willing to learn, but first must be presented with good arguments. Einstein is repeatedly impressed how much progress experimental physics has made. But that the STR and GTR have passed all experimental tests does not surprise him one bit ...

Following the confirmation by Eddington of his predicted value of light deflection at the solar limb Einstein was transformed by the press virtually overnight into a celebrity of the first order. Occasionally, he enjoyed this role, but it was mostly just annoying to him. He has demonstrated that himself with different comments:

> "To punish me for my contempt of authority, Fate has made me an authority myself." [17-9]

> “In the past it never occurred to me that every casual remark of mine would be snatched up and recorded. Otherwise I would have crept further into my shell.” [17-15]