Spiral Galaxy NGC 3190
(FORS/VLT)

ESO Press Photo 17/06 (11 May 2006)

D Lorentz Transformations, Velocity Addition and Doppler Effect

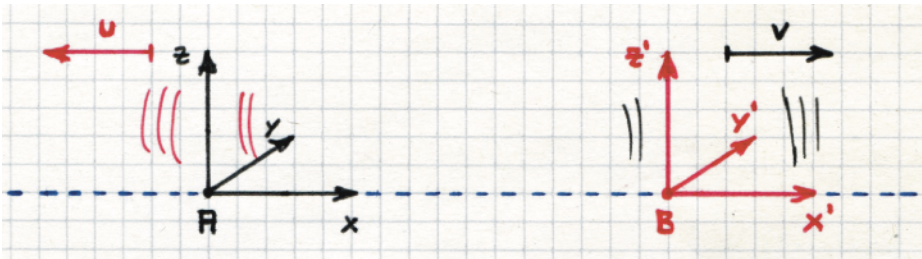
We examine the transformation of coordinates between different inertial frames in the mechanics of Galileo and Newton and deduce the old formula for the addition of velocities. Then we do the same considering the 3 fundamental effects of STR. We deduce the so-called Lorentz transformations twice: First with assistance of Epstein diagrams and a second time only using the formulas from section **B**. From the Lorentz transformations we obtain the relativistic formula for the addition of velocities. Finally we deduce the optical Doppler formula.

D1 Coordinate Transformations before the STR

A physical *event* takes place at a certain time and at a certain place. Thus, in each inertial frame (coordinate system, reference frame) we can assign to an event a time coordinate as well as three local coordinates. In each coordinate system one point of 4D space-time belongs to an event.

We examine in this and the following two sections how the 4 coordinates assigned by an observer to an event in one inertial frame are correctly converted to the corresponding 4 coordinates assigned by a second observer of the same event in another inertial frame. The formulas describing this conversion are called *coordinate transformations*.

We assume (as previously) two coordinate systems - a black, non-prime system and a red, prime system, which are aligned to each other in a simple manner (same diagram as in **B3**):



The origin B of the red system moves with velocity v along the x-axis of black, and the x'-axis of red coincides with the x-axis of black. Thus A moves with velocity u = -v along the x'-axis of red. The two other spatial axes (y/y' and z/z') will always be parallel to each other. In addition, both red and black set their clocks to zero at the moment when they coincided. All other clocks that black possibly uses are synchronized within its frame with the master clock in A. And all 'red' clocks are synchronized with the red clock B within the red system.

We now consider coordinate conversions within the mechanics of Galileo and Newton: Since the clocks were set to zero by red and black at their meeting, both clocks (as well as all other clocks of red and black) will always agree. They agree to the degree of their accuracy, velocity of movevment has no influence on the synchronization or on the clock tick rate. Thus for each moment and in all places it applies

(1) $t = t'$

Also concerning the distances from the x-axis, which is identical to the x'-axis, one will find no differences. That is

(2) $y = y'$ and $z = z'$

There is something to convert only if one wants to convert an x'-coordinate of red to x or vice-versa. x' is the distance from the point of the event, projected on to the x/x' line, to the red origin B. Origin B has the distance $v \cdot t$ from the black origin A. Thus the x-coordinate of the event is calculated simply as

(3) $x = x' + v \cdot t = x' + v \cdot t'$ and correspondingly $x' = x - v \cdot t$

We have just derived the very simple Galileo transformations. We have crucially made use of Newton's concept of absolute time, which is the same for everyone, as well as his concept of absolute space, which permits the calculation of absolutely valid distances or lengths.

We will now demonstrate (as promised in **A3**) that the 'classical' addition of velocity results from this basis of Galileo and Newton ('classic' is always equivalent to 'not relativistic' in this context).

Assume C moves in the red system with the speed of w' in the x'-direction. The x'-coordinate of C is thus $x' = a + w' \cdot t'$, where a is any constant. This is the distance in the x'-direction from B to C. B is however at any given time t, as seen from black, at the point $v \cdot t$. The point of C along the x-axis of black is thereby $x = v \cdot t + a + w' \cdot t'$. We thus have already used the absoluteness of space. Now we use Newton's absolute time and simply replace t' by t using (1). Thus we obtain
$x = v \cdot t + a + w' \cdot t = a + (v + w') \cdot t$.
So the x-position of C increases with speed v + w' for black. Speeds simply add in classical mechanics.

In order to compare with the somewhat more complicated Lorentz transformations, which we will deduce in the next section, we present the Galileo transformations in the following table:

$$\boxed{\begin{aligned} t' &= t \\ x' &= x - v \cdot t \\ y' &= y \\ z' &= z \end{aligned}} \qquad \boxed{\begin{aligned} t &= t' \\ x &= x' + v \cdot t' \\ y &= y' \\ z &= z' \end{aligned}}$$

These coordinate transformations describe how in classical mechanics the coordinates (t, x, y, z), which black attributes to an event should be converted into the coordinates (t', x', y', z'), which red attributes to the same event - and vice-versa. If the two coordinate systems were less precisely aligned to each other, then naturally the lines 2, 3 and 4 in the small boxes would be somewhat more complicated. Nothing at all would change however in the first line, which reflects Newton's absolute time.

A somewhat childish suggestion: Go against the official direction of motion on an escalator (as you surely once did as a child) or on one of the long "moving sidewalks" one finds in airports. It is fun and also a direct way to experience the addition (or rather subtraction) of velocities.

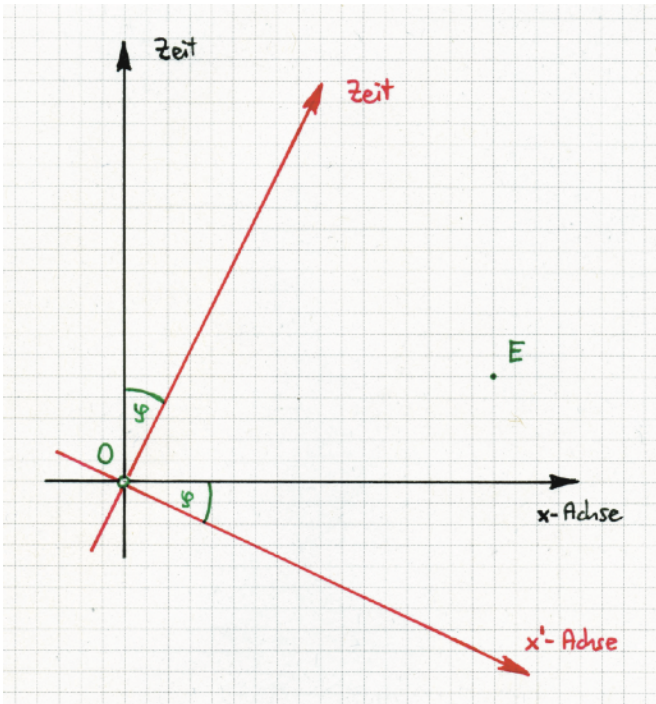
D2 Derivation of the Lorentz Transformations from Epstein Diagrams

If black and red want to compare their measured values for place and time which they assign to an event, then it presupposes that they have already crossed paths and synchronized their clocks. Synchronization means that both set their “master clocks” at this meeting at $x = 0 = x'$ to $t = 0 = t'$ and synchronized any other clocks within their system with the master clock. To compare the coordinates of an event is always to talk about a *second* contact of red and black. Otherwise the measured values of red and black would be arbitrary! This remark by the way applies to all space-time diagrams, not only to those of Epstein.

Consider two coordinate systems meeting at O as described in the preceding section. Now a red clock moves by a particular location of black, and the coordinates (t', x', y', z') are recorded. Which coordinates (t, x, y, z) will black attribute to this meeting? How can we, in general, convert such event coordinates, given that we account for the relativistic effects of time dilation, length contraction and desynchronization derived in section **B** from Einstein's basic postulates?

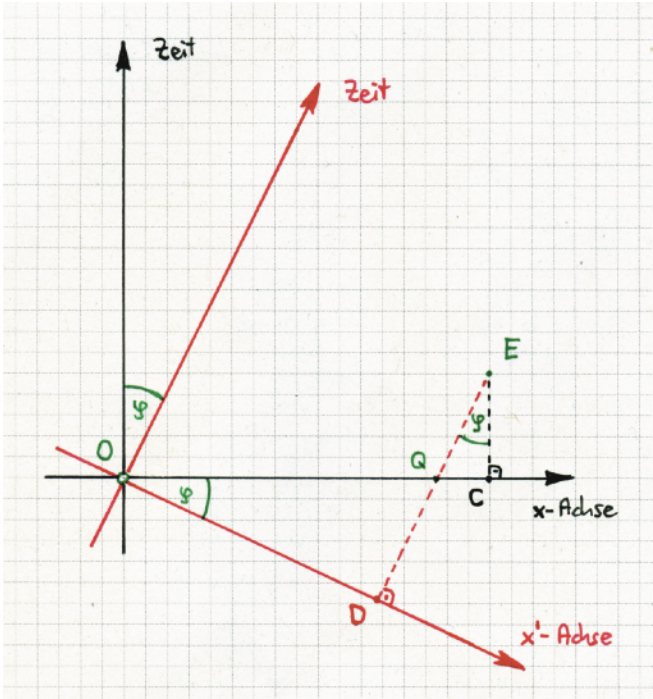
This question is answered by the Lorentz transformations already mentioned in **A3** and **A4**. In this section we derive the Lorentz transformations from Epstein diagrams. For skeptics a second derivation follows in the next section based only on the three basic phenomena and their quantitative description in section **B**.

First we note that the convenient equations $y = y'$ and $z = z'$ still apply. As described in **B3** distances perpendicular to the relative velocity of the two systems, that is, perpendicular to x and x', are the same for both black and red. We need only worry about the time coordinates and the spatial coordinates in the direction of relative motion v. And it is exactly these values which are perfectly represented in the Epstein diagram. We start with a nearly ‘empty’ Epstein diagram:

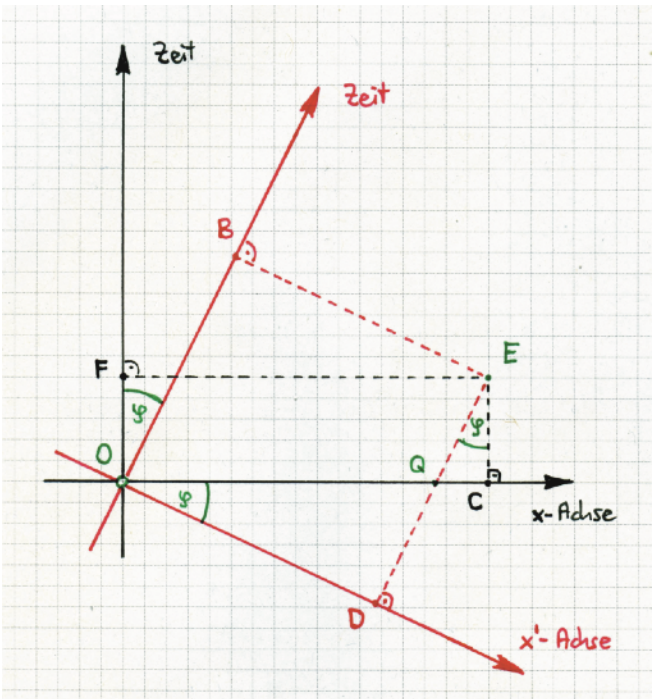


We mark an arbitrary point E in space-time. No doubts exist about where E lies in black and red: We need only project E onto the x-axis, respectively x'-axis. We obtain the points C and D (in the following figure). These projections additionally yield the auxiliary point Q, which we will later make use of. We have

(1) $x = OC$ and $x' = OD$ and still (2) $y = y'$ and $z = z'$



E defines the event “the red clock of x' is at location x of black”. The time t' of this event can only be measured with the red clock which is locally present! Now we draw the projections on the two time axes and obtain the points F and B:

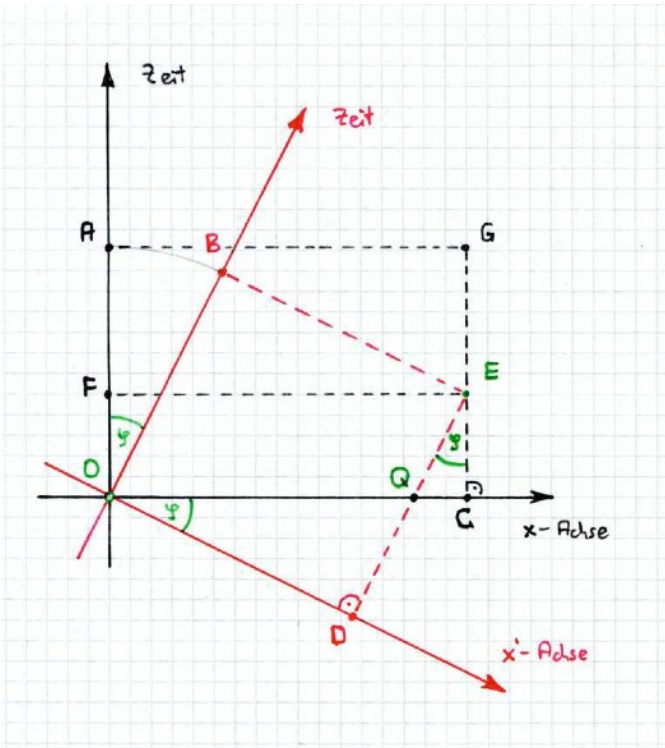


For red the clocks at O and D (or, somewhat later, at B and E) are synchronized. Black sees things differently. The time indicated by the local red clock for event E, is for black simply

(3) t’ = OF

Thus we have accounted for the two effects of time dilation and desynchronization!

The time t of black is still missing in the diagram. Which clock does black use to measure the flyby of the red clock at location x? Clearly it uses the one at location x, that is, the one which was at point C when the meeting of the master clocks took place at point O. However, where is this clock, given that the red clock has moved from D to E? According to the dogma of Epstein everything moves equally far through space-time between two events. The black clock at location x is therefore at point G, and therefore we have by necessity CG = OA = OB = DE.



The two points E and G in the Epstein diagram *both* belong to the event “the red clock in its system at location x', flies past location x of black” ! This is often confusing for those, who are well versed with space-time diagrams for which an event always corresponds to a *single* point in the diagram.

So black attributes the following time coordinate to this event:

(4) t = OA = OB = DE

We still have to find the values (t, x, y, z) for the associated values (t', x', y', z') and vice-versa. We already know how it is with y and z. We now investigate how to obtains the values (t, x) from (t', x'). The reverse transformations will be left as a small algebra exercise. We use lengths for all space-time distances and must therefore multiply time values by the speed of light c. Thus:

$t \cdot c = OA = OB = DE = DQ + QE = OD \cdot \tan(\phi) + CE / \cos(\phi) = x' \cdot \sin(\phi) / \cos(\phi) + t' \cdot c / \cos(\phi)$

Recalling the meaning of sin(ϕ) and cos(ϕ), we are already finished:

$t \cdot c = x' \cdot (v / c) / \sqrt{} + t' \cdot c / \sqrt{} = (t' \cdot c + x' \cdot v / c) / \sqrt{}$

Dividing by c and writing the whole somewhat more conventionally, yields

$$t = \frac{t' + \frac{x'}{c} \cdot \frac{v}{c}}{\sqrt{1 - \frac{v^2}{c^2}}}$$

The two terms in the numerator beautifully represent (together with the denominator) the effects of time dilation and desynchronization. For x' = 0 we have only the first effect whereas t' = 0 underscores the second effect.

Just as easily we can derive how one obtains x from (t', x'):

x = OC = OQ + QC = OD / cos(ϕ) + EC • tan(ϕ) = x' / cos(ϕ) + t' • c • sin(ϕ) / cos(ϕ) thus

x = x' / √ + (t' • c • v / c) / √ = (x' + v • t') / √

Here the difference to the corresponding Galileo transformation is less significant, showing as it were only length contraction.

Here are the resulting transformations:

$$t' = \frac{t - \frac{x}{c} \cdot \frac{v}{c}}{\sqrt{1 - \frac{v^2}{c^2}}}$$
$$x' = \frac{x - v \cdot t}{\sqrt{1 - \frac{v^2}{c^2}}}$$
$$y' = y$$
$$z' = z$$

$$t = \frac{t' + \frac{x'}{c} \cdot \frac{v}{c}}{\sqrt{1 - \frac{v^2}{c^2}}}$$
$$x = \frac{x' + v \cdot t'}{\sqrt{1 - \frac{v^2}{c^2}}}$$
$$y = y'$$
$$z = z'$$

The transformations (t, x, y, z) ↔ (t', x', y', z') presented here must mutually cancel, if they are executed consecutively. This calculation is recommended to the reader as an exercise.

As author of this book I would like to sing the highest praise for the Epstein diagram. In the USA one would call me an 'evangelist'. I am indeed quite proud to be the first Epstein evangelist to show the derivation of the Lorentz transformations from an Epstein diagram …

For the skeptics or otherwise incorrigible non-believers a derivation of the transformations follows in the next section which is completely devoid of Epstein diagrams and based only on the results of section **B**.

D3 Derivation of the Lorentz Transformations from Basic Phenomena

Consider again two coordinate systems, as described at the beginning of **D1**. An event E takes place for red at time t' at the point (x', y', z'). We now know that it takes place for black at the point (x, y, z) with y = y' and z = z'. Still black's values for t and x are to be determined for this event.

For black the 'master clock' of red at location B (like each clock of red) runs too slowly, thus

$t = t(A) = t'(B)/\sqrt{}$ or $t \cdot \sqrt{} = t'(B)$

A clock at location x' of red, in addition, shows for black a desynchronization to the clock at B of

$\Delta t' = -x' \cdot v/c^2$

Thus the red clock at B already shows $t'(B) = t'(x') - \Delta t' = t' + x' \cdot v/c^2$, when E takes place. Thus we obtain the corresponding clock state for A from the expression

$t(A) \cdot \sqrt{} = t'(B) = t' + x' \cdot v/c^2$

For t = t(A) itself we obtain $t = (t' + x' \cdot v/c^2) / \sqrt{}$

It was precisely this expression for t that we derived in **D2**!

We still must clarify, at which x-coordinate the event E takes place for black. Red thinks that the distance d' from A to the location x' of the x'-coordinate of E has the following value:

$d' = x'(E) - x'(A) = x' + v \cdot t'(B) = x' + v \cdot t'(x') = x' + v \cdot t'$

Here we have used the fact that for red the clocks at B and at x' are synchronized. Thus for red the event E has the distance d' from A along the x-axis, as well as along the x'-axis. For black all measurements of red in the x-direction are Lorentz contracted. The distance of the event from A must therefore be for black $d = d'/\sqrt{}$, and we are finished:

$x = d = d'/\sqrt{} = (x' + v \cdot t') / \sqrt{}$

It was precisely this expression for x that we also derived in **D2**!

It is interesting to note that most high school text books about the STR do not *derive* these Lorentz transformations. Often they are 'assumed' or simply 'stated' and then time dilation and length contraction are derived from them. The reverse path from the basic phenomena to these more abstract transformations can be taken, only after desynchronization has been quantitatively dealt with.

The reason for introducing the Lorentz transformation is however the same with all authors: They provide a simple derivation of the correct formula for the addition of velocities. Here we have to deal in principle with three inertial frames: B moves with v relative to A, and C moves with w' relative to B. What speed does C then have for A? The angle ϕ between the time axes can be the same, e.g. 60°, for both A and B and for B and C. If one combines rather naively the two corresponding Epstein diagrams into one with 3 time axes, then the angle between the time axes of A and C is already 120°! Even I, as Epstein evangelist, was not spared until recently the derivation of the formula for the addition of velocities in the STR via the Lorentz transformations.

Alfred Hepp made me aware of the fact that Epstein shows in the second edition of [15] in appendix A, how one can construct with compass and straight-edge the correct tilting angle for the speed of w from those for v and w'. Thanks to the sketch in that section (which also first requires understanding!) we could finally find a quite simple proof for the addition formula (red box on the following page), which is based only on Epstein diagrams, and with which one can completely avoid Lorentz transformations. I present this proof in the appendix **K7**.

D4 Addition of Velocities in the STR

In **D1** we showed that the addition of velocities is simple in the mechanics of Galileo and Newton: If B moves with velocity v in the x-direction of A and C moves for B with velocity w' in the same direction, then C moves with velocity v + w' for A. We recognized in **A3** that this simple formula cannot apply in the STR: The light of a locomotive standing still must travel forward just as fast as that of one moving forward, i.e. with c.

For the new velocity addition formula it must be true that the sum of v and c results in c. It must also be true that in no case may a velocity be greater than c. If a spaceship flies past us at 0.7 • c and fires off a rocket in the direction of its flight, which itself has velocity 0.8 • c relative to the spaceship, then we would already have a speed of the rocket of 1.5 • c according to Newton …

The derivation of the correct formula for the addition of velocities is quite harmless, if the Lorentz transformations are available:

Let C move with velocity w' in the x'-direction of B, while B moves as usual with a relative velocity v along the x-direction of A. Then for the x'-coordinate of C we have

$x' = a + w' \cdot t'$ where a is some constant

We now simply substitute both x' and t' by expressions with x and t from the Lorentz transformations:

$x' = (x - v \cdot t) / \sqrt{}$ and $t' = (t - x \cdot v / c^2) / \sqrt{}$

Thus the equation from above becomes

$(x - v \cdot t) / \sqrt{} = a + w' \cdot (t - x \cdot v / c^2) / \sqrt{}$

Multiplying both sides by the radical we obtain

$x - v \cdot t = a \cdot \sqrt{} + w' \cdot (t - x \cdot v / c^2)$ or

$x = a \cdot \sqrt{} + v \cdot t + w' \cdot (t - x \cdot v / c^2) = a \cdot \sqrt{} + v \cdot t + w' \cdot t - w' \cdot x \cdot v / c^2$

From this we obtain

$x + x \cdot w' \cdot v / c^2 = a \cdot \sqrt{} + v \cdot t + w' \cdot t$ or $x \cdot (1 + w' \cdot v / c^2) = a \cdot \sqrt{} + (v + w') \cdot t$

Dividing by the bracketed term on the left we obtain

$x = a \cdot \sqrt{} / (1 + w' \cdot v / c^2) + (v + w') / (1 + w' \cdot v / c^2) \cdot t$

Since both the radical and the bracketed term are constant we can read from this that C moves for A with the constant velocity of

$w = (v + w') / (1 + v \cdot w' / c^2)$

along the x-axis!

If we use the symbol ⊕ to represent the relativistic addition of speeds that are parallel to the relative velocity v, then we can summarize:

$$\boxed{v \oplus w' = \frac{v + w'}{1 + \frac{v \cdot w'}{c \cdot c}}}$$

With the symbol + we denote the 'usual' addition of numbers. In the numerator we have the usual addition of speeds, while the denominator provides for corrections, as soon as the values of v/c or w'/c become substantial. For small speeds of v and w' the denominator is practically 1.

In exercise 5 we check that this formula supplies reasonable values in all cases. In the above example of the spaceship with its rocket we get a resulting velocity of

$0.7 \cdot c \oplus 0.8 \cdot c = (1.5/1.56) \cdot c \approx 0.962 \cdot c$

D5 Transverse Velocities and Aberration

How is it for black, when in the red system an object moves with a velocity of u' *transverse* to the direction of relative motion v ? Up to now we have considered only movements along the x-axis.

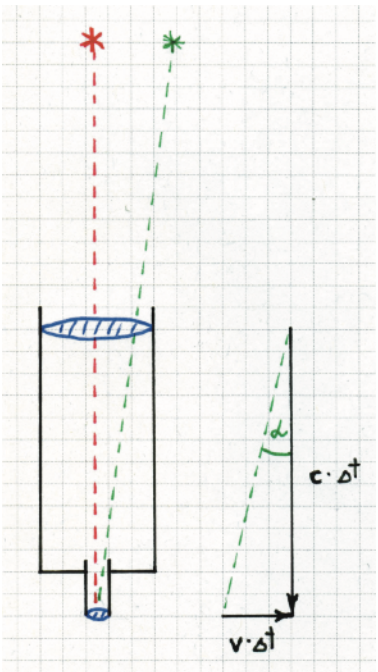
For the derivation of the transformation of such 'transverse velocity' we do not need the Lorentz transformations. Knowledge of the basic phenomena is completely sufficient. Since u' is a velocity e.g. in the y'-direction of red, it follows

$u' = \Delta y' / \Delta t' = \Delta y / \Delta t' = \Delta y / (\Delta t \cdot \sqrt{}) = (\Delta y / \Delta t) / \sqrt{} = u / \sqrt{}$

Black thus measures the smaller lateral velocity $u = u' \cdot \sqrt{}$

We will need this result in **E1**. We use it here for the derivation of the correct formula for aberration. By aberration (lat. aberrare ~ wander, deviate) we understand the change of direction of velocities, which arise as a result of the fact that the viewer likewise moves. James Bradley recognized in 1728 that the tiny annual-periodic position shifts of fixed stars are to be understood as the consequence of the movement of the earth around the sun. According to legend he got the idea, as he rode in his coach in windless rainy English weather and thereby observed that the rain seemed to fall diagonally, the faster the coach moved the more diagonal the rain.

Consider a telescope, pointing in a direction perpendicular to the momentary direction of the earth's movement in its orbit:



In the time the light of a star needs to arrive from the objective to the eyepiece, the earth has already advanced on its course. We must therefore tip the telescope through an angle α, in order for the star to be presented in the center of the visual field. This angle defines the amount 'the ray of light wanders off' due to the movement of the earth. The resulting formula for this aberration is $\tan(\alpha) = v/c$, where v is the velocity of the earth in its orbit (approximately 30 km/s). The angle has a size of approximately 20 arc seconds.

Einstein had already in 1905 drawn attention to the fact that this traditional formula is only approximately correct. Lateral velocities should be transformed according to the above formula, resulting in the correct formula
$\tan(\alpha) = v/(u' \cdot \sqrt{}) = v/(c \cdot \sqrt{})$ One also obtains this formula, if one assigns (correctly) the distance travelled by the light to the hypotenuse of the triangle instead of the leg in the above figure. For the lateral velocity u of the light, the Pythagorean Theorem gives $u = c \cdot \sqrt{}$, which again yields for $\tan(\alpha)$ the value $v/u = v/(c \cdot \sqrt{})$. Thus for light the new accurate aberration formula is $\sin(\alpha) = v/c$. This correction is astronomically insignificant, since the values of the sine and the tangent functions hardly differ for small angles.

By the way: The angle of aberration is independent of the speed of light in the telescope tube. The angle of aberration does not change if you fill your telescope with water! A more in depth argument would consider the direction of the optical wave planes.

We could now consider the general case, where in the red system an object moves with any velocity w' + u' in any direction. Einstein already handled this case in his original publication [09-140ff] and presents beautiful symmetrical formulas for the resulting velocities and angles in the black system. Likewise the aberration is treated completely generally [09-146ff]. The appropriate calculations should now be well comprehensible to the reader. We will however not need these results in what follows. **K3** makes some references to it.

D6 The Optical Doppler Effect

You all know the phenomenon: An ambulance approaches at high speed with howling siren. As it races by the pitch of the siren sinks and remains constantly at a deeper level as it departs. Also the other situation is well well-known, where you yourself move at high speed past a standing source of noise: You are travelling in your car over country, windows open, and you past a train crossing with alarm bells.

These changes in the perceived pitches correspond to measurable changes in the frequencies of the acoustic waves. Christian Doppler examined this theoretically and concluded that the two cases “listener moves, source at rest” and “source moves, listener at rest” must differ. In 1842 he ascribed formulas, indicating how the measured values of the frequencies and wavelengths change. Today one calls the phenomenon in his honor the “Doppler effect”. We can easily understand why one may not only consider the relative motion of source and listener *acoustically*: The sound spreads out homogeneously with a certain speed in all directions *in the medium of air* ! This carrier medium supplies a special inertial frame and naturally served as the model for the ether, in which the light should spread.

Consider an observer B with velocity v approaching a resting acoustic source Q. This produces a tone of frequency f(Q). What frequency f(B) does the observer measure? Doppler's answer to this question is the following, whereby here c means the speed of sound in air:

(1) $f(B) = f(Q) \cdot (1 + v/c)$ Observer approaches a resting acoustic source

For the case where the acoustic source approaches with velocity v an observer at rest in the medium air, we have

(2) $f(B) = f(Q) / (1 - v/c)$ Acoustic source approaches a resting observer

For values of $v/c < 0.1$ the two results hardly differ. The difference becomes arbitrarily large however as v/c approaches the value 1.

We change now from sound to light or more generally to electromagnetic waves. The STR precludes the possibility of determining absolutely, who is at rest and who is moving. Thus ‘optically’ there is only one Doppler formula! We deduce it first from (2):

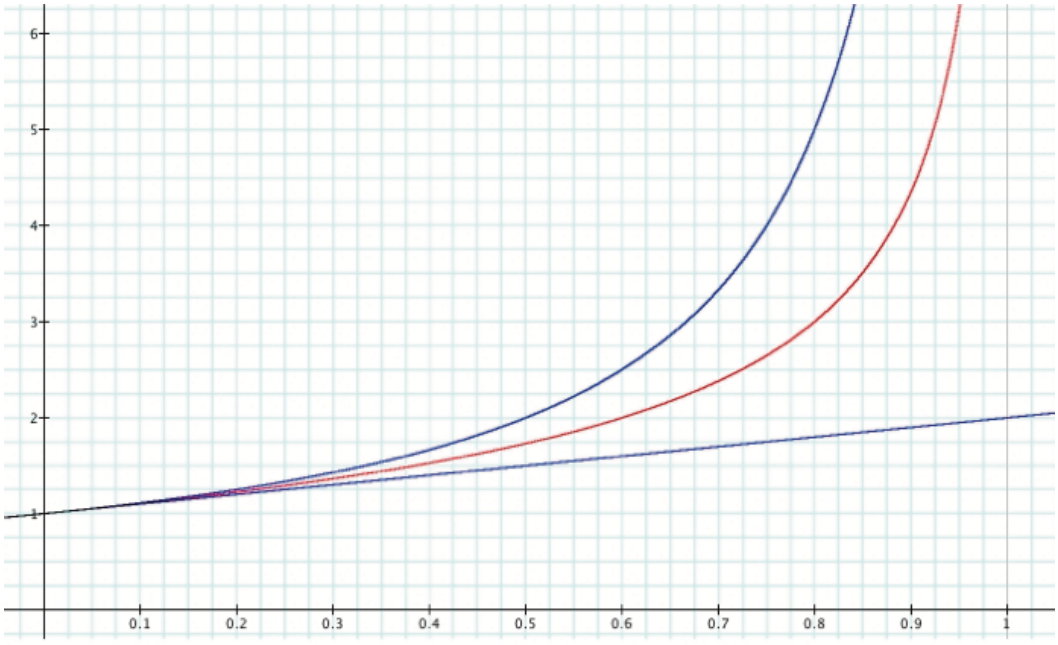
Formula (2) remains valid, but we must now consider additionally that the oscillator of the transmitter, because of time dilation, oscillates for the observer B only with the frequency $f(Q) \cdot \sqrt{}$! Thus we have $f(B) = f(Q) \cdot \sqrt{} / (1 - v/c)$. Considering that we can write $\sqrt{} = \sqrt{} ((1 + v/c) \cdot (1 - v/c))$, we can reduce a little obtaining

$$\boxed{f(B) = f(Q) \cdot \sqrt{\frac{1 + v/c}{1 - v/c}} = f(Q) \cdot \sqrt{\frac{c + v}{c - v}}}$$

We obtain the same result, if we argue from the view of the source at rest and proceed from formula (1). The receiver B then counts more oscillations with his slowed clock, i.e. $1/\sqrt{}$ times as many per second as someone whose clocks do not tick more slowly. Thus $f(B) = f(Q) \cdot (1 + v/c) / \sqrt{}$, which after simplifying provides the same result above.

Both Doppler formulas provide for light in the STR the same frequency shift and the cases ‘source at rest’ and ‘observer at rest’ can no longer be differentiated.

Let us graph the three functions $y = 1 + x$; $y = 1/(1 - x)$ and $y = \sqrt{((1 + x)/(1 - x))}$ with the value of $x = v/c$ ranging over the unit segment 0 to 1. We have the following picture:



The lower, linear function belongs to Doppler formula (1), the upper blue to Doppler formula (2). The middle red curve describes the optical (or relativistic) Doppler effect in accordance with the formula deduced previously. The differences begin to be evident only when v/c is larger than about 0.2. Starting from a value of v/c greater than 0.5 the differences become increasingly dramatic.

In astronomy the optical Doppler effect has important applications. Frequencies of spectral lines, however, are not measured but rather the wavelengths (usual symbol λ). Therefore we should transform the above formula accordingly:

In general $\lambda \cdot f = c$ or $f = c / \lambda$. Thus

$c / \lambda(B) = (c / \lambda(Q)) \cdot \sqrt{((c + v) / (c - v))}$ and after division by c

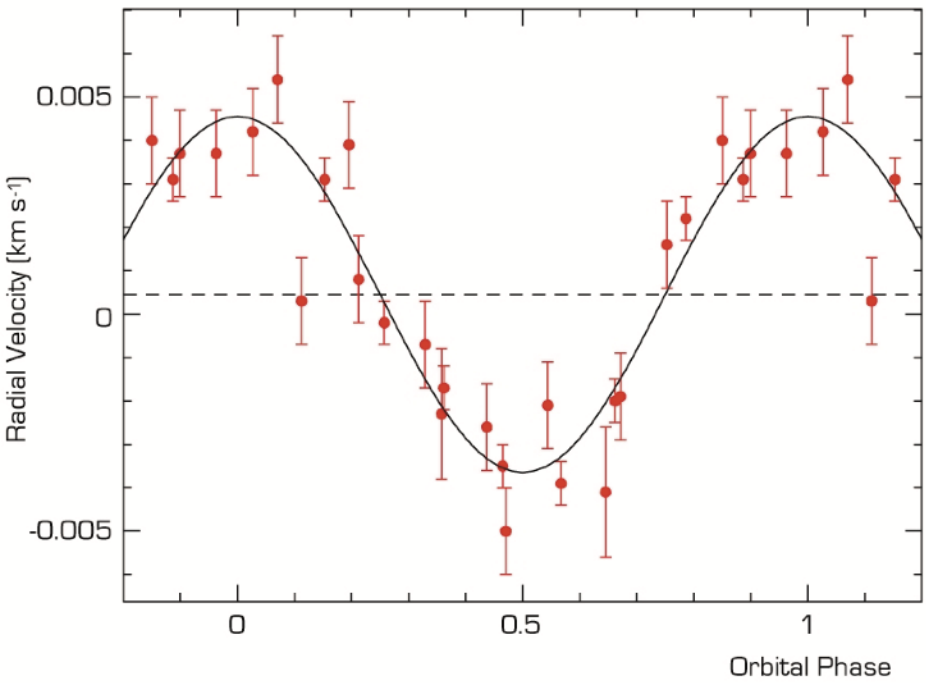
$\lambda(Q) = \lambda(B) \cdot \sqrt{((c + v) / (c - v))}$ or $\lambda(B) = \lambda(Q) \cdot \sqrt{((c - v)/(c + v))}$

λ(Q) is well-known and λ(B) is measured. From this the velocity v can be computed, with which the source moves toward us ($v > 0$) or away from us ($v < 0$). This is the so-called radial velocity. If one solves the above formula for v, then one obtains

$$v = c \cdot \frac{\lambda(Q)^2 - \lambda(B)^2}{\lambda(Q)^2 + \lambda(B)^2}$$

In the last few years astronomers have developed such precise spectrometers that they can measure periodic fluctuations in the radial velocity of stars within the range of a few meters per second. This has become one of the most important methods for showing the existence of planets orbiting other stars (so-called exoplanets). The graphic below gives an impression of the precision that has been obtained. The measured values of the radial velocity have an uncertainty of approximately ± 1 m/s! These fluctuations of the radial velocity result from the fact that both the planet *and* the star orbit a common center of mass.

Further information is freely available on the well maintained web page of the ESO. The following graphic was taken from the ESO press release of August 25, 2004. Consult the web-site for an answer to the question, how long does an 'orbital phase' last, in other words, how long is the period of this planet in days or hours.



"Velocity Curve" of mu Arae

ESO PR Photo 25d/04 (25 August 2004)

http://www.eso.org/outreach/press-rel/pr-2004/pr-22-04.html

D7 Problems and Suggestions

1. A fighter jet flies with 1000 m/s and shoots a projectile off in its flight direction with a muzzle velocity of likewise 1000 m/s. Add these velocities ‘classically’ and ‘relativistically’.

2. Derive the Lorentz transformations for t' and x' algebraically from those for t and x, which we deduced first in **D2** and then a second time in **D3**.
Why is that actually unnecessary?

3. Show algebraically that the Lorentz transformations from the non-prime to the prime system and vice-versa mutually cancel each other.

4. Derive the Lorentz transformations for t' and x' from an Epstein diagram!

5. Examine our formula from **D4** for relativistic velocity addition. Are $v = 0.5 \cdot c$, $w = 0.8 \cdot c$, $u = -0.5 \cdot c$ and c all parallel velocities. Form a) $v \oplus v$ b) $v \oplus w$ c) $v \oplus c$ d) $c \oplus w$ e) $c \oplus c$ f) $c \oplus u$ g) $u \oplus -c$ h) $w \oplus w$

6. How quickly is a star approaching us, given that the Hα line for an excited hydrogen atom is not found to be 656 nm as in a laboratory on earth, but rather at 649 nm? (The emission line is thus a little ‘blue’ shifted).

7. How fast does one have to approach a traffic light, so that one sees the red light (wavelength of 620 nm) as green (wavelength of 520 nm)?

8. A laser produces light at 632 nm wavelength. What wavelength do we measure, if this laser is at the tail of a UFO, which is moving away from us at $0.5 \cdot c$?

9. Why does the rotation of a star show up as a widening of its spectral lines?

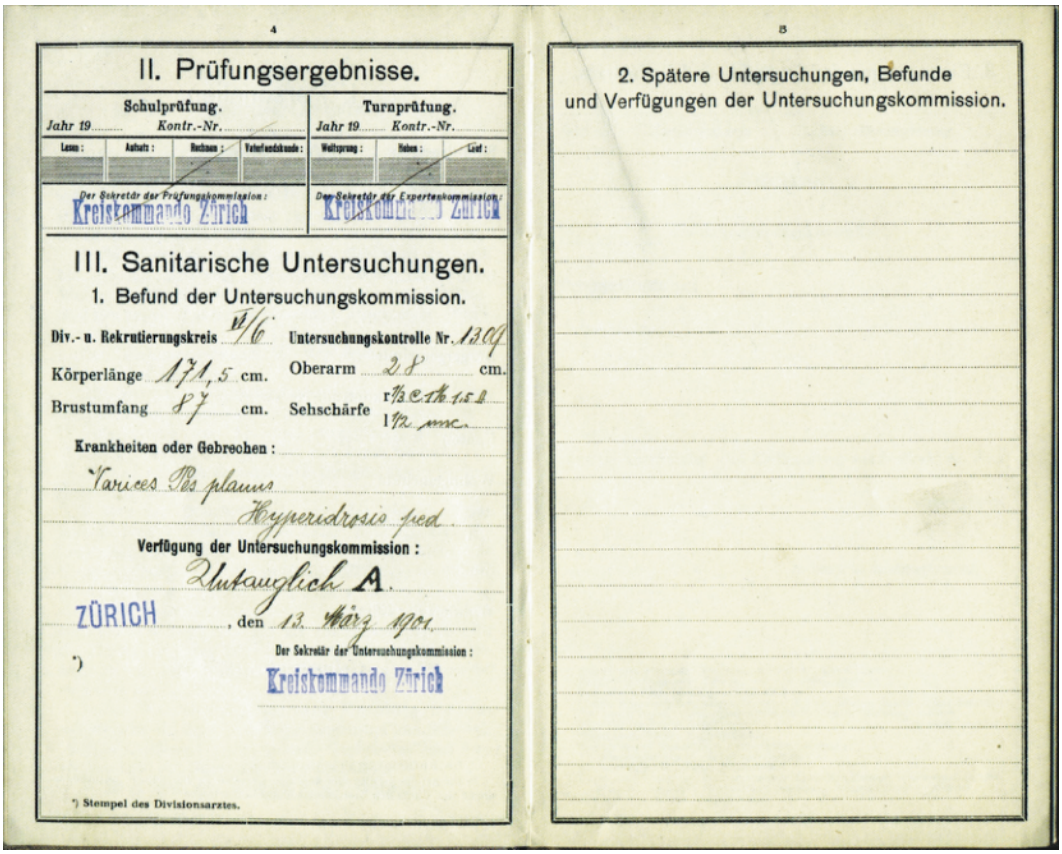
10. Why do spectral lines widen, if an emitting gas exhibits a high temperature and high pressure? (The effects in problems 9 and 10 express themselves quantitatively differently and can be partly computationally separated, if they arise superimposed.)

11. Derive the optical Doppler formula from the acoustic Doppler formulas for the wavelengths, by additionally considering length contraction!

12. Read pages 140-142 as well as 146-149 from Einstein's original publication in [09].

13. In addition to our 2 coordinate systems (black and red with the points A and B) there is a ‘middle’ system C, in which A moves equally fast to the left as B to the right. Ascertain, in general, the velocity of this middle system C for both A and B. Without STR the answer would naturally be v/2 and -v/2…
The existence of this middle system C, by the way, provides a beautiful argument for the fact that the relative velocities of B for A and of A for B must be quantitatively equal: From the point of view of C the situation is perfectly symmetrical!

14. How do the radar speed measurements of the traffic police function? Consider the whole from the frame of the reflecting car.



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II. Prüfungsergebnisse.

Schulprüfung. Jahr 19..... Kontr.-Nr.....

Lesen: | Aufsatz: | Rechnen: | Vaterlandskunde:

Der Sekretär der Prüfungskommission: Kreiskommando Zürich

Turnprüfung. Jahr 19..... Kontr.-Nr.....

Weitsprung: | Heben: | Lauf:

Der Sekretär der Expertenkommission: Kreiskommando Zürich

III. Sanitarische Untersuchungen.

1. Befund der Untersuchungskommission.

Div.- u. Rekrutierungskreis VI/6 Untersuchungskontrolle Nr. 1304

Körperlänge 171,5 cm. Oberarm 28 cm.

Brustumfang 87 cm. Sehschärfe r 1/2 c th 15 l 1/2 mc

Krankheiten oder Gebrechen:

Varices Pes planus

Hyperidrosis ped.

Verfügung der Untersuchungskommission:

Untauglich A.

ZÜRICH, den 13. März 1901.

Der Sekretär der Untersuchungskommission:

Kreiskommando Zürich

*)

*) Stempel des Divisionsarztes.

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2. Spätere Untersuchungen, Befunde und Verfügungen der Untersuchungskommission.

Albert Einstein's Swiss Military "Service Booklet". He was declared exempt from service when found to have flat and sweaty feet - certainly to his great satisfaction.

Einstein fled from Munich not only because he found the prevailing spirit at the Luitpold Gymnasium unbearable. He also feared being conscripted by the military - an idea that certainly filled him with horror and persuaded him to abandon German citizenship. He deeply hated everything militaristic, and repeatedly supported conscientious objectors and lobbied his entire life for disarmament and the strengthening of supranational institutions.

> "This topic brings me to that worst outcrop of the herd nature, the military system, which I abhor. That a man can take pleasure in marching in formation to the strains of a band is enough to make me despise him. He has only been given his big brain by mistake; a backbone was all he needed. This plague-spot of civilization ought to be abolished with all possible speed." [20-6]

Yet Einstein was no naive pacifist. In view of what was brewing in the early thirties in Germany on his Berlin doorstep, he forsook his previous strictly pacifist line and wrote to a Belgian military objector:

> "Organized power can be opposed only by organized power. Much as I regret this, there is no other way." [17-168]