





Spiral Galaxy Messier 83 (VLT ANTU + FORS1)

ESO PR Photo 41/99 (29 November 1999)

© European Southern Observatory

## B The Three Fundamental Consequences

Adhering to the principle of relativity and Maxwell's equations has three fundamental consequences: The simultaneousness of two events loses their absolute character, different observers of an activity will no longer measure the same duration, and the distance between two points (or the length of an object) loses its absoluteness. We discuss the experimental confirmation of these basic phenomena from two perspectives. Finally we derive the amount the two clocks appear desynchronized to a moving observer, given that they are synchronized in their own inertial frame.

## B1 Primo: The Relativity of Simultaneity

Much ink has been spilled concerning the nature of 'time'. Einstein's break-through insight however sounds quite banal: Time is what one reads from a nearby clock:

### Time is what one reads from a local clock

Deep insights are often not at first sight perceptible as such ...

First we want to convince ourselves that it is possible to synchronize several identical clocks which are at rest in an inertial frame at different places. Often the following method is suggested: two clocks are at points A and B respectively. A flash of light is released at the midpoint of AB and on arrival of the light each clock is set to 0000 and started.

The problem is: how does one synchronize a third clock C with clock A without losing the synchronization between A and B? And isn't finding the midpoint already a problem? This 'standard method' is actually unworkable.

It is however quite possible to synchronize as many clocks as desired with a given clock A: The 'master' clock A emits a flash of light at an arbitrary but well-known time  $t_0$ . As soon as the light arrives at clock B, it is firstly reflected, secondly B's clock is set to 0000 and thirdly it is started. Clock A records time  $t_1$ , when the light reflected from B arrives again at A. One calculates the elapsed time  $(t_1 - t_0) / 2$  for the light from A to B, records the value  $t_0 + (t_1 - t_0) / 2$  and sends it by snail mail to B. The (continuously running) clock B is then *advanced* by this value. One does not need the midpoint AB at all and in addition one obtains the distance between the two clocks.

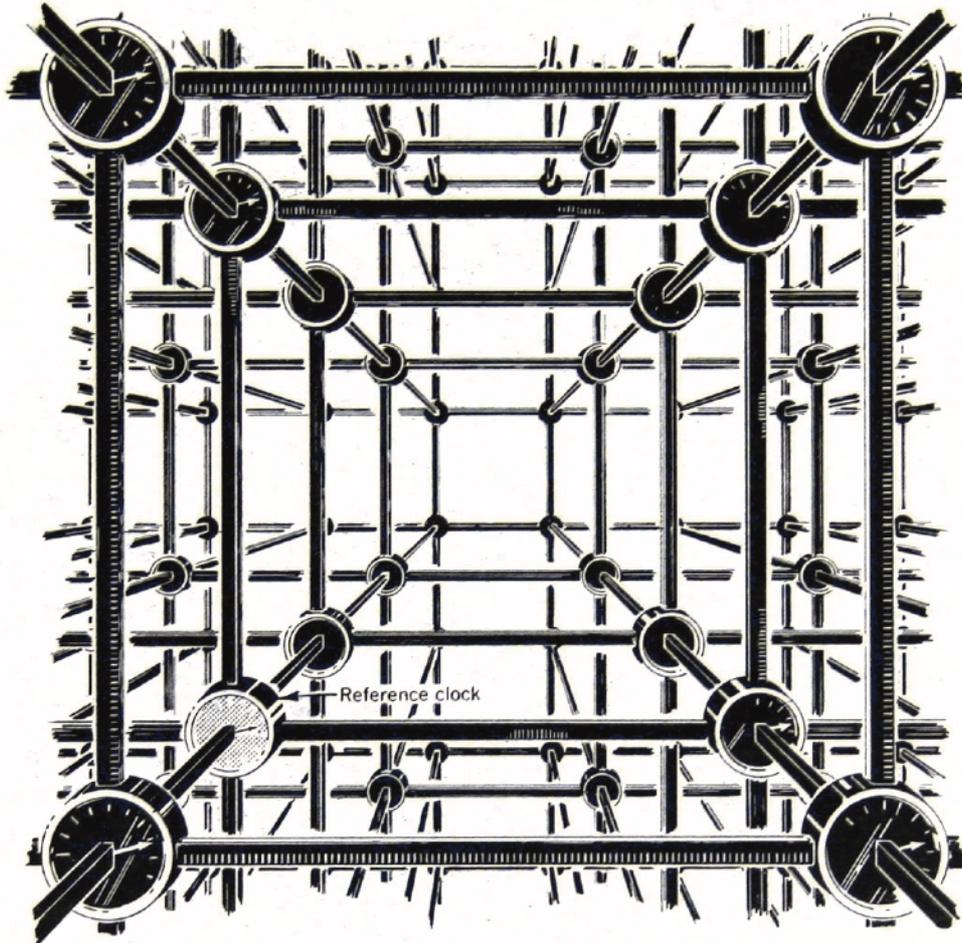
Hans Reichenbach pointed out in different publications starting from 1920 that this definition implies a further assumption, i.e. the isotropy of space (a collection of Reichenbach's early writings on space, time and motion in English translation has been edited by Steven Gimbel and Anke Walz in [12]). In particular the speed of light should be equal in all directions. Measuring the one-way speed of light presupposes distant clocks which have already been synchronized. Therefore the synchronization of distant clocks and the measuring of the one-way speed of light have a circular relationship to each other. When we computed the elapsed time for the light from A to B as  $(t_1 - t_0) / 2$  we tacitly assumed that the light needs equal time to travel in both directions! The postulate of isotropy was hidden in this assumption. The book "Concepts of Simultaneity" by Max Jammer [13-218] presents two simple axioms which a set of clocks must meet, in order to be synchronizable. The formulation of the first axiom is ours:

1. If a clock A sends out two light signals with  $\Delta t_A$  time difference, then each further clock B must receive the signals with  $\Delta t_B$  time difference, where  $\Delta t_B = \Delta t_A$ .
2. The time required for light to traverse a triangle is independent of the direction taken around the triangle.

The first axiom must surely be fulfilled if synchronized clocks are to remain synchronized. Obviously it can be fulfilled only by clocks which are at rest relative to each other! The second axiom (called the "round trip axiom") guarantees that the speed of light is independent of direction. Taken together the two axioms are necessary and sufficient so that a set of clocks can be synchronized.

Since we will need the postulate of isotropy of space in **B3**, we introduce it here to the STR. Its operational formulation concerning the speed of light is found in the "round trip" axiom.

Thus, in an inertial frame one can have clocks at arbitrary locations, which are all synchronized in the sense described above. Measuring the point of time of an event means to read the time from such a synchronized clock positioned at the location of the event. Thus we now have a conception in terms of a hardware view of an inertial frame, as represented in [14-37]:



Copyright © 1992 by Edwin F. Taylor and John Archibald Wheeler

What does one see in this scaffolding of clocks on the dial of one of the distant clocks?

For all physical quantities the following three aspects can never be completely disassociated:

- a) the definition of the quantity
- b) the method of measurement of this quantity and
- c) the definition of its units of measurement.

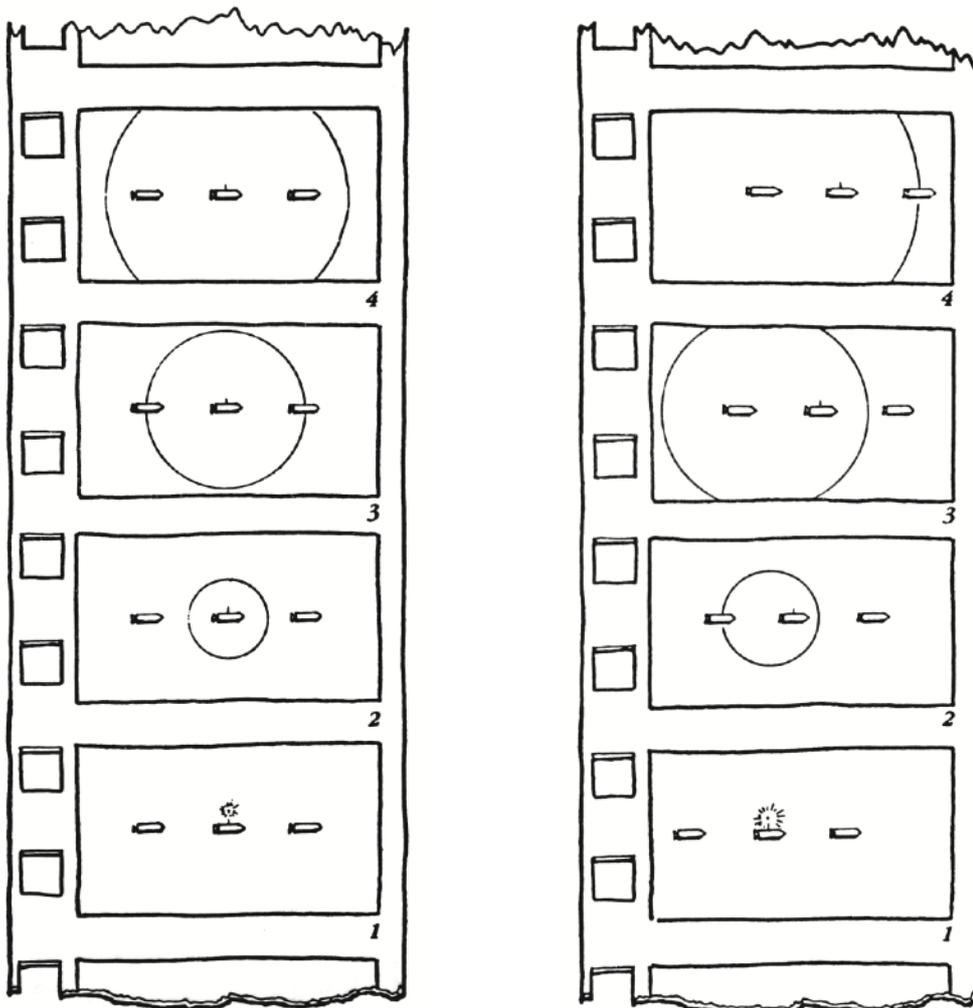
To define 'time' as a physical quantity means to construct some copies of an accurate clock and to define how to compare the times indicated by these clocks (consult the Wiki articles for the second and for the international atomic time !).

Einstein's insightful contribution to the development of the STR was to show that this operational approach eliminates all difficulties.

Thus in an inertial frame we can get along with one single time. We are allowed to speak of *the* time  $t$  in an inertial frame. However, different inertial frames usually have different chronologies for events. Epstein demonstrates this very beautifully in [15-33ff]. He considers an interstellar fleet of three spaceships, which travel in a row in space at a constant distance from each other:



There is an inertial frame, in which the three spaceships are at rest. A radio call from the flagship will reach the other two spaceships at the same time in this inertial frame (movie strip on the left, should be read from the bottom to the top). On the other hand, for a viewer in an inertial frame in which the fleet moves, the radio call reaches the leading spaceship later than it does the following ship (movie strip on the right)! This is a direct consequence of the fact that the radio signal in each inertial frame travels in all directions with the constant speed  $c$ .



Copyright © 2000 Lewis C. Epstein, *Relativity Visualized*

Quantitatively however the strip on the right is not completely correctly drawn. Here  $c$  appears to be somewhat larger than for the one on the left, and above all the fleet does not move with uniform speed, which the reader can easily verify with a ruler.

Given the postulate that  $c$  has the same constant value in every coordinate system and that light or radio signals spread in each inertial frame with the same speed into all directions in space, it follows immediately that it makes sense only within an inertial frame to say that two events take place at the same time. Poincaré had already pointed out in 1902 :

"The English teach mechanics as an experimental science; on the Continent it is taught always more or less as a deductive and a priori science. The English are right, no doubt. How is it that the other method has been persisted in for so long; how is it that Continental scientists who have tried to escape from the practice of their predecessors have in most cases been unsuccessful? On the other hand, if the principles of mechanics are only of experimental origin, are they not merely approximate and provisory? May we not be some day compelled by new experiments to modify or even to abandon them? These are the questions which naturally arise, and the difficulty of solution is largely due to the fact that treatises on mechanics do not clearly distinguish between what is experiment, what is mathematical reasoning, what is convention, and what is hypothesis. This is not all.

1. There is no absolute space, and we only conceive of relative motion; and yet in most cases mechanical facts are enunciated as if there is an absolute space to which they can be referred.

2. There is no absolute time. When we say that two periods are equal, the statement has no meaning, and can only acquire a meaning by a convention.

3. Not only have we no direct intuition of the equality of two periods, but we have not even direct intuition of the simultaneity of two events occurring in two different places. I have explained this in an article entitled "La Mesure du Temps".

4. Finally, is not our Euclidean geometry in itself only a kind of convention of language? Mechanical facts might be enunciated with reference to a non-Euclidean space which would be less convenient but quite as legitimate as our ordinary space; the enunciation would become more complicated, but it still would be possible.

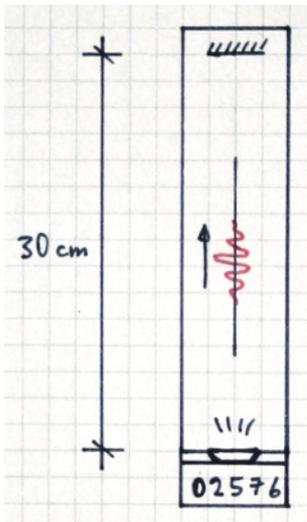
Thus, absolute space, absolute time, and even geometry are not conditions which are imposed on mechanics. All these things no more existed before mechanics than the French language can be logically said to have existed before the truths which are expressed in French. We might endeavour to enunciate the fundamental law of mechanics in a language independent of all these conventions; and no doubt we should in this way get a clearer idea of those laws in themselves." [16-89ff]

Einstein and his friends Solovine and Habicht (from right to left) carefully studied Poincaré's book at the "Akademie Olympia". With his operational definitions Einstein analyzed the "conventions", which permit us to speak about simultaneousness within an inertial frame. Just as clearly he showed that clocks which are synchronized in one frame do not run synchronously when observed from another moving frame. Also the amount of desynchronization was given an exact numerical value. We address this quantitative aspect in **B6**.



## B2 Secundo: Fast Clocks Tick More Slowly

It gets even worse: not only does the concept of simultaneousness become meaningless when we observe events from two separate inertial frames which are moving with respect to each other, but the river of time itself flows at different speeds! For the derivation of this difference we need only the Pythagorean Theorem.

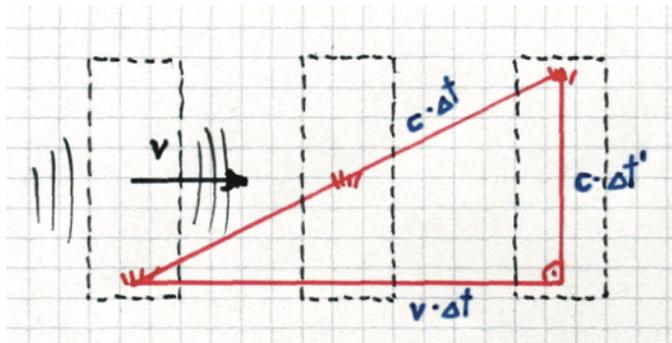


We assume the constancy and universality of  $c$  and consider “light clocks” with the following design:

In a pipe a flash of light travels from the bottom up to a mirror at the top, where it is reflected (“tick”) back to the bottom. There it is sensed by a photoelectric cell (“tock”) releasing a new flash and incrementing a counter by 2. The counter can be read at any time. Take a moment and consider why the clock should be 30 cm in length and why the counter is increased each time by 2.

Imagine that we have several such clocks and that some are set up and synchronized along the  $x$  axis of our coordinate system at known distances. A further identically constructed clock moves with speed  $v$  past these “resting” clocks (only this “fast” clock is shown below at three positions in the diagram!). How much time elapses in the resting system, during the time the “fast” clock makes a single “tick”?

The distance light travels in the moving clock (call it the prime system with measured time  $t'$ ) is 30 cm or, in general,  $c \cdot \Delta t'$ . But what is the distance this light travels as seen from the resting system (call it the non-prime system with measured time  $t$ ) and in relation to which the moving clock travels along the  $x$ -axis with velocity  $v$ ? Given the constancy of the speed of light this will be, of course,  $c \cdot \Delta t$ . These two distances are however not equal and thus the time intervals,  $\Delta t$  and  $\Delta t'$ , must differ! The Pythagorean Theorem provides us with the relationship between these two values:



Obviously more time elapsed in the resting, non-prime-system, than in the moving, prime-system, since the light travelled a longer distance in that system. Thus:

$$(c \cdot \Delta t)^2 = (v \cdot \Delta t)^2 + (c \cdot \Delta t')^2 ; c^2 \cdot (\Delta t)^2 = v^2 \cdot (\Delta t)^2 + c^2 \cdot (\Delta t')^2 ; (\Delta t')^2 = (\Delta t)^2 \cdot (1 - v^2/c^2)$$

and we get

$$\Delta t' = \Delta t \cdot \sqrt{1 - \frac{v^2}{c^2}}$$

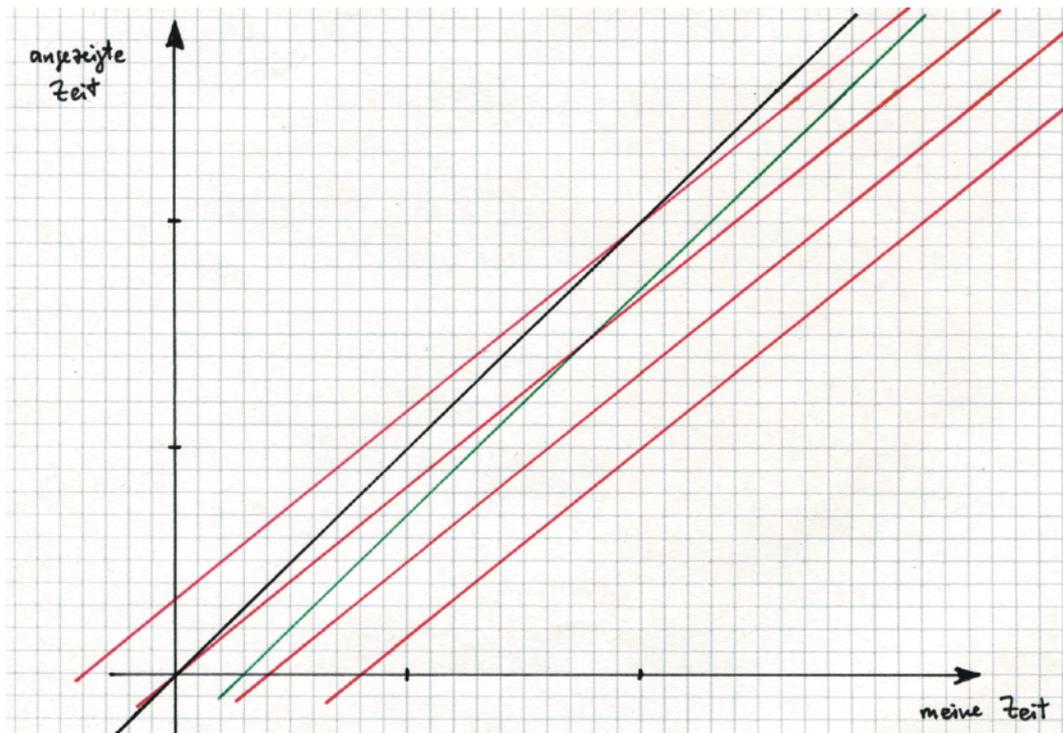
Thus moving clocks tick more slowly, compared to a set of clocks at rest. One calls this effect 'time dilation'. Search the Internet with keywords 'light clock' or 'time dilation' and you will find innumerable nice animations concerning this consequence of holding to **M** and **R**.

In the prime system we measure the *proper time* of a "tick". A proper time interval is measured by a single clock between two events that occur at the same place as the clock. In the non-prime system we need two synchronized clocks to measure the time interval corresponding to a single "tick" of the moving clock.

The proper time interval is always the *longest* time interval measured by a single clock between two events, that mark the beginning and the end of a given process! Section **H4** is entitled with "The Principle of Maximum Proper Time (Eigenzeit)". From the point of view of the prime system, the non-prime clocks are moving, and all of them tick more slowly than its own clock which is at rest. So each of those clocks would measure a shorter time interval for the same process. How do we explain however that we just measured a longer duration in the non-prime system? Does this not contradict our principle of maximum proper time? Perhaps you already see how to resolve this apparent contradiction: The important fact is that in order to make the measurement in the non-prime system, we need at least two distant clocks...

We will clarify this point completely towards the end of section **B6**!

Thus little remains of Newton's absolute time! It makes sense to draw a diagram showing the time indicated by identical, perfect (!) clocks in relationship to the time indicated on my perfect clock:

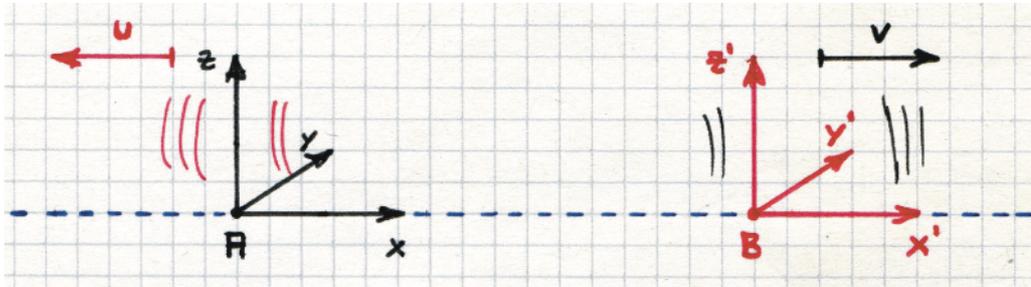


black: my clock and all other perfect clocks synchronized with it in my inertial frame  
green: a correct, but badly synchronized clock at rest in my inertial frame  
red: "fast" clocks, which can be synchronized in their own inertial frame

Try adding to the diagram an imperfect, but more or less well synchronized clock at rest. Also add a second clock at rest, which is synchronized at time point 10, but then runs fast!

### B3 Tertio: Moving Yardsticks Are Shorter

The inertial frame B (prime, red) moves with constant speed of  $v$  along the  $x$  axis of the inertial frame A (non-prime, black):



The STR should be consistent in the following sense: both A and B make the same statements about which time intervals or lengths in A and B are measured. They will not measure the same values, but they both can figure out, what the other one measured, and agree about these values. We draw from this fact the following important conclusion: If B moves for A with velocity  $v$  in the positive  $x$ -direction, then A moves for B with the velocity  $-v$  in the  $x'$ -direction! In addition to the speed of light  $c$  both also have the amount of their relative velocity in common. Most authors assume that this is self-evident. Is it really?

We consider what alternatives might be possible: Assume that B measures a relative velocity  $u$  of the two systems where  $|u| < |v|$ . In this case A also knows that B measures a smaller relative velocity. If space is isotropic (looks the same in all directions) and the STR is consistent as described above, then the situation is perfectly symmetrical. In this case B will correspondingly state that A measures a smaller relative velocity. Thus we have a contradiction: it follows for the magnitudes of the relative velocities that  $v < u < v$ , which is impossible. Therefore B can measure neither a smaller nor larger relative speed of the two systems than A, it must be that  $u = -v$  and  $|u| = |v|$ . Here again (remember **B1** !) we need the postulate of isotropy!

System A has 2 synchronized clocks at a distance  $\Delta x$  from each other. B moves with relative velocity  $v$  over this distance and measures with its clock the time  $\Delta t'$ , which elapses between the meetings with the two clocks of A. B uses  $\Delta t'$  and  $v$  to determine the distance between the two clocks in system A:  $\Delta x' = v \cdot \Delta t'$ .

But what does A observe? A measures  $\Delta t$  between the two clock meetings and the distance  $\Delta x$  of its clocks and determines the speed  $v$  of B:  $v = \Delta x / \Delta t$ . The value of  $v$  is the same for A and B, and so we have the following equation

$$\frac{\Delta x}{\Delta t} = v = \frac{\Delta x'}{\Delta t'} \quad \text{and thus} \quad \Delta x' = \Delta x \cdot \frac{\Delta t'}{\Delta t} = \Delta x \cdot \sqrt{1 - \frac{v^2}{c^2}}$$

using the result of the last section. The distance  $\Delta x$  which is at rest in the system A appears in system B to be shortened by the same factor we have already encountered.

We have preferentially treated the  $x$ -direction (which corresponds to the  $x'$ -direction and the direction of the relative velocity); actually we know only that lengths of moving objects in the direction of relative motion are shortened. What is the behavior in perpendicular directions?

Consider Einstein's train, moving with speed  $v = 0.6 \cdot c$  in the x-direction on a long straight railroad line. If the train has the length 300 m in its own reference system, then we will measure it at a shortened length of 240 m (do the math!). Did the train also become narrower? If so then it would have fallen between the rails (which are at rest) when it reached a certain speed. That would, in principle, be possible. However, if moving objects would contract themselves perpendicular to the direction of motion, then from the point of view of the travelers in the train it would mean that the separation between the moving rail tracks became smaller! And the theory would require the track width to be too large and too small at the same time - impossible! Therefore we conclude that there is no 'lateral contraction'.

In summary:

Moving objects appear shortened in their direction of motion, that is, the length of an object measured at rest is always the longest. We call this the principle of *maximum proper length*. (Note: curiously google comes up empty in a search for this expression whereas the principle of maximum proper time is well known.) Perpendicular to the direction of the relative motion the measured values of all observers agree. The following formulas apply:

$$\Delta x' = \Delta x \cdot \sqrt{1 - \frac{v^2}{c^2}}$$

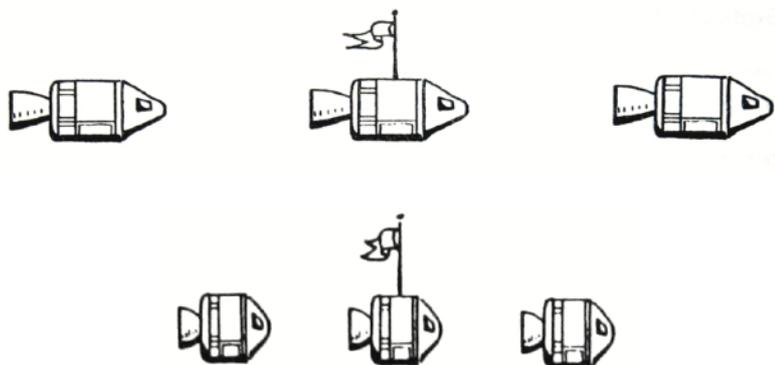
$$\Delta y' = \Delta y$$

$$\Delta z' = \Delta z$$

In addition the value of the radical agrees for both reference systems, since in both the square of the relative velocity has the same value.

By the way, the absence of lateral contraction is quite significant for our argument in **B2**! Otherwise the path of the light perpendicular to  $v$  would not be of equal length in both systems, and we would not have a unique length of the vertical leg of the triangle. The perpendicularly standing light clock only becomes more narrow and not shorter or longer. We were indeed fortunate...

Thus Epstein's small fleet, if at first it is at rest and then speeds past an observer at very high speed, appears so (Illustrations [15-39f]):



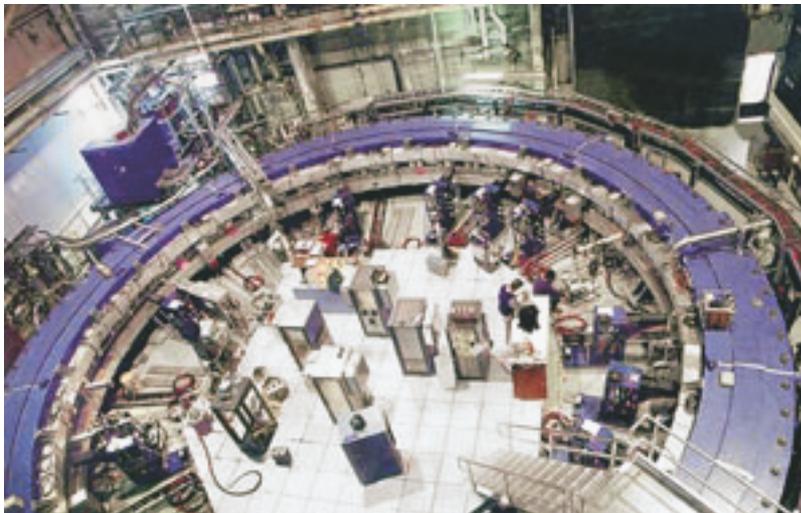
Copyright © 2000 Lewis C. Epstein, *Relativity Visualized*

## B4 Muons 1: An Experimental Confirmation

For a long time there existed hardly any experimental confirmation to Einstein's relativity theories. This changed dramatically with the development of atomic clocks and modern electronics starting around 1960, providing a boost to relativity theory as an area of research.

A first confirmation of relativistic time dilation was found in the extended half-life of fast-moving muons (B.Rossi and D.B.Hall 1941). These come into being at a height of 10 to 20 km over the earth's surface, when high-energy particles of cosmic radiation strike atoms in the terrestrial atmosphere. Muons differ from electrons in that they have a much larger mass and are unstable. Slow muons have an average life span or 'half-life' of  $1.52 \mu\text{s}$ . The extremely fast muons, which are produced by the cosmic radiation, move nearly at the speed of light and should therefore according to Newton travel about  $1.52 \cdot 10^{-6} \cdot 3 \cdot 10^8 \text{ m}$ , approximately 456 m, during their life time. The flow of muons should therefore be halved, if one decreases the altitude by 456 m. However it actually decreases much more slowly. Since the muons are created at an altitude of about 15 km, they must travel 33 times the distance 456 m to reach sea level. Out of  $2^{33}$  muons only one should reach our detector. However this does not fit the observed density of the muon stream: in Germany approximately 200 muons per square meter per second are counted at sea level.

CERN scientists tested time dilation of muons quantitatively much more exactly in 1975. Large quantities of muons were produced and captured with 99.942% the speed of light in a special storage ring. It showed that their half-life at this speed amounts to  $44,6 \mu\text{s}$ , in complete agreement with our formula of **B2** (do the math!). More details are provided in [11-13f].



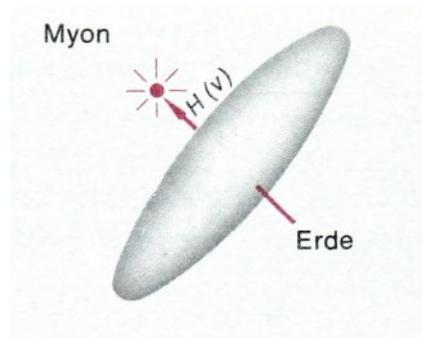
Muon storage ring at the Brookhaven National Laboratory, USA

## B5 Muons 2: A Second Look

Let's again consider the muons produced at an altitude of 15 km by cosmic radiation. Such a muon is at rest in its own inertial frame and thus has the usual half-life of  $1.52 \mu\text{s}$ . This small half-life should only be enough for the earth to approach the muon by around 456 m. Why then do so many muons experience the arrival of the earth's surface?

Time dilation is no help here. But the 15 km thickness of the atmosphere extends in the direction of the earth's motion towards the muon and thus appears to the muon 'Lorentz contracted'. Using the formula of **B3** we calculate for  $v = 0.99942 \cdot c$  a root factor of 0.03405. For our muon the 15 km diminishes to 511 m! The earth traverses these 511 m in somewhat more time than the half-life of a muon, i.e. nearly half of all muons experience the arrival of the earth's surface.

Seen by the muon the earth keeps its old cross sectional area. The earth's diameter and also the thickness of its atmosphere shrink however in the direction of motion down to 3.4% of the rest value. The earth thus takes on the shape of a flat disk. [11-14f] offers no derivation of the length contraction, but it indicates in the text and in one of the many border illustrations (which make the book so attractive) that this contraction offers an explanation in the muon's inertial frame. The border illustration on [11-15], shown below, contains however 2 errors, a harmless one in the picture and a worse one in the text. Can you find the two errors?



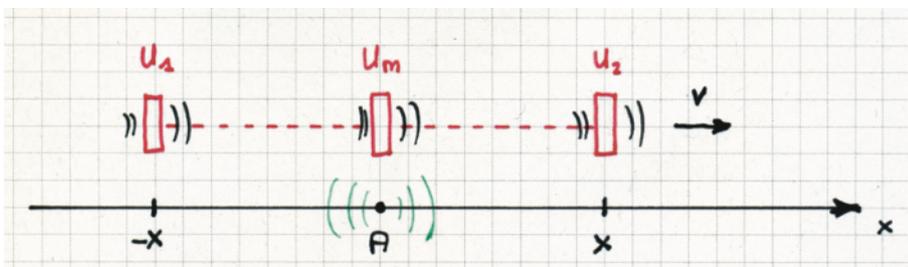
*Seen from the point of view of the muons, the earth which is approaching them at nearly the speed of light, appears greatly flattened. All distances in the direction of motion are shortened. The muons are thus able to travel the distance to the earth's surface within their life time.  
(translation by Samuel Edelstein)*

In both representations (earth at rest with moving muons and muon at rest with approaching earth) one arrives at the same conclusion about the portion of muons which collide with the earth's surface. The reasoning is however completely different. The 'history' each tells differs strongly from that of the other one. 'History' is truly 'his story'... (wordplay personally communicated to me by Floyd Westermann). This however results in no contradictions or conflicts concerning the physics.

## B6 Quantitative Aspects of the Relativity of Simultaneity

In **B1** we concluded that the synchronization of a set of clocks in different inertial frames must fail. Even if the synchronization were possible for a given time point it would make little sense considering **B2**. It is however possible to exactly say by how much 2 clocks which are synchronized in the red system B are desynchronized from the point of view of the black system A. We will now derive this formula. It appears only in a few books concerning STR. It is however indispensable, if one really wants to put all the pieces together. This will be clear from the set of example problems presented at the end of this section.

We introduce three clocks  $U_1$ ,  $U_m$  and  $U_2$  moving in relationship to each other like Epstein's small fleet, that is, at constant distance from each other with velocity  $v$  in the  $x$ -direction of the black, at rest, non-prime system A.  $x'$  is the distance of neighboring clocks in the red, fast-moving, prime system B;  $x$  is the corresponding value measured in the black system.  $x'$  is larger than  $x$ .



At the exact point when  $U_m$  flies past the zero point A of the black system, a flash of light is released. We call this time point 0. In the red system the two clocks  $U_1$  and  $U_2$  are thereby synchronized ( $U_m$  remains in the middle of  $U_1$  and  $U_2$  whether or not Lorentz contraction takes place!). However when are  $U_1$  and  $U_2$  for the black system triggered by this flash of light?

$U_1$  is flying toward the flash;  $U_1$  will encounter the flash at time point  $t_1$  yielding  $t_1 \cdot c = x - t_1 \cdot v$ ; and thus  $t_1 = x / (c + v)$

$U_2$  is flying away from the flash;  $U_2$  will encounter the flash at time point  $t_2$  yielding  $t_2 \cdot c = x + t_2 \cdot v$ ; and thus  $t_2 = x / (c - v)$

The clock in front,  $U_2$ , will be started for the black system with the following delay:

$$t_1 - t_2 = \dots \text{ (do the math!) } \dots = -2 \cdot v \cdot x / (c^2 - v^2)$$

That is the time difference for the black system, which knows however that the red clocks run more slowly than its own. We discover the time difference of the red clocks only if we multiply this value by our radical:

$$\Delta t' = (t_1 - t_2) \cdot \sqrt{\dots} \text{ (do the math!) } \dots = -2 \cdot x \cdot (v/c^2) / \sqrt{\dots}$$

$2 \cdot x / \sqrt{\dots}$  is exactly the distance  $\Delta x'$  of the clocks  $U_1$  and  $U_2$  as measured in their own system! Thus we have the quite simple result

$$\Delta t' = -\Delta x' \cdot \frac{v}{c^2}$$

The red clocks are synchronized in their own system and have distance  $\Delta x'$  in the direction of the relative motion. However, these clocks are desynchronized in the black system by the amount  $\Delta t'$ . One can write the result differently, seeing more clearly that the formula is as simple as possible:

$$\Delta t' \cdot c = -\Delta x' \cdot \frac{v}{c}$$

The factor  $c$  on the left of the equals sign serves only to convert times into lengths. The de-synchronization is thus proportional to the distance between the red clocks in the direction of motion and also to the ratio  $v/c$ .

That one cannot do without this formula, if one wants to present the whole situation without contradiction, will be clarified through a careful study of the following

### Sample Problem

A particle moves with  $v = 0.8 \cdot c$  through a 12 m long pipe, which is equipped with detectors at both ends, which in turn contain clocks, allowing one to measure the transit flight time precisely. The pipe is at rest in the black system. Let the particle's rest system be the red system. We pose and then answer the following questions:

1. How long does the transit flight of the particle through the pipe last for black?
2. How much time elapses thereby in the red system (of the particle) from the point of view of the black?
3. How long is the pipe for red?
4. How long does it last for red, until the pipe has raced over the particle?
5. How much time elapses from the point of view of red for this flyby on each clock of black?
6. How does red explain the measured value of black??

The questions 5 and 6 are omitted in most text books, whereby they form the capstone of understanding of the STR.

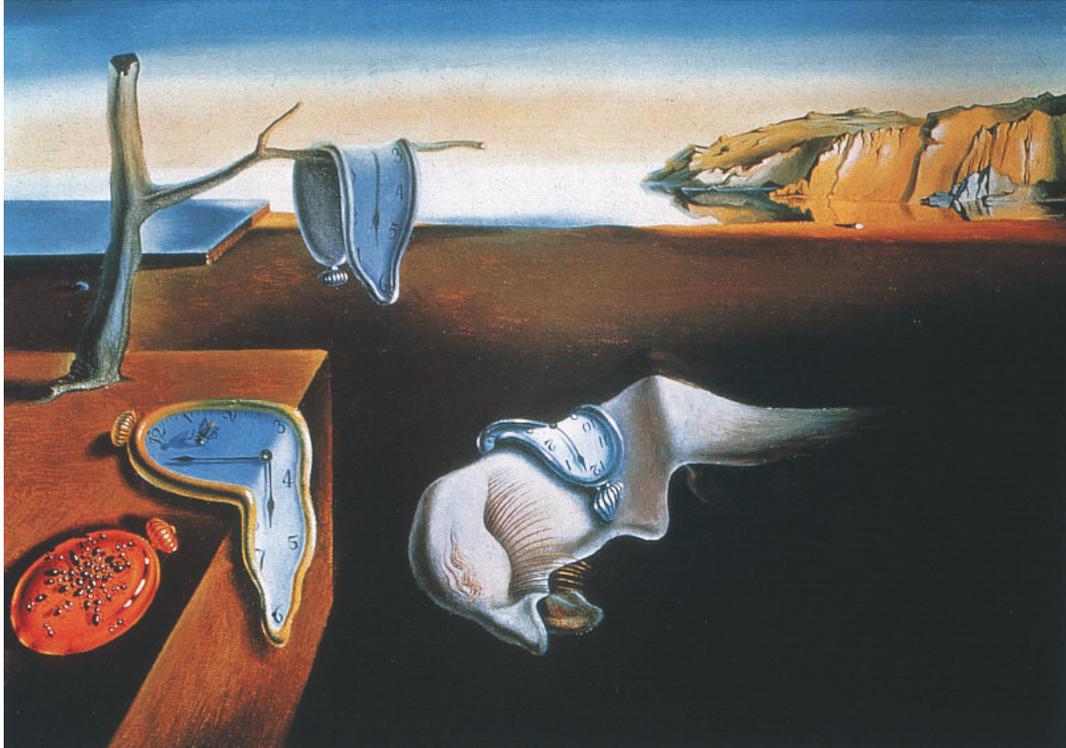
### The answers:

1. Time is distance divided by velocity:  $\Delta t = \Delta x/v = 12 \text{ m} / (0.8 \cdot 3 \cdot 10^8 \text{ m/s}) = 50 \text{ ns}$
2. Because of the time dilation red will measure a shorter duration:  
 $\Delta t' = \Delta t \cdot \sqrt{v} = 50 \text{ ns} \cdot 0.6 = 30 \text{ ns}$
3. Red sees the pipe as Lorentz-contracted:  $\Delta x' = \Delta x \cdot \sqrt{v} = 12 \text{ m} \cdot 0.6 = 7.2 \text{ m}$
4. Until the 7.2 m long pipe has completed flying over red:  
 $\Delta t' = \Delta x'/v = 7.2 \text{ m} / (0.8 \cdot 3 \cdot 10^8 \text{ m/s}) = 30 \text{ ns}$  (in complete agreement with black!)
5. The fast-moving clocks of black tick more slowly for red than they do in their own frame. Thus the flyby in the black system as seen by red lasts only  $\Delta t = \Delta t' \cdot \sqrt{v} = 30 \text{ ns} \cdot 0.6 = 18 \text{ ns}$  ( !! )
6. Also red knows that black measured 50 ns, however red attributes it to the fact that the two clocks of black are desynchronized by  $\Delta t = \Delta x \cdot v/c^2$  which numerically constitutes exactly  
 $12 \text{ m} \cdot 0.8 / (3 \cdot 10^8 \text{ m/s}) = 32 \text{ ns}$ . The 18 ns duration plus 32 ns desynchronization yield together the 50 ns that black measured with his two clocks!! Check that the sign is also correct.

## B7 Problems and Suggestions

1. Compute the root term for different values of  $v$ : For cars, airplanes, rockets etc. What does it mean for Newton's absolute time, when one considers only 'earthly' speeds?
2. How quickly does one have to move, in order that "one hour takes only 3599 s"?
3. How long would the running of '24 hours of Le Mans' last for the drivers, if they drive on average (somewhat exaggerated) at 324 km/h? The answer may depend on the pocket calculator that is used...
4. How far does light actually travel in a nanosecond? What distance corresponds to this precisely measurable time interval?
5. Provide a table of the distances of the planets from the sun, measured in 'light minute' units.
6. Two rockets fly past each other at  $0.6 \cdot c$ . A measures the length of the other rocket B to be 40 m. What is the rest length of the rocket B, and how much are the clocks at the tip and at the end of rocket B for A desynchronized, given that they are synchronized for B? And which of the two clocks is running fast for A?
7. How fast does a clock have to move, in order to halve its running-time?
8. Signal travel times: The singing of Mick Jagger is broadcast directly from the microphone to a radio listener 300 km away. The listener sits 6.8 m from the loudspeaker. How long does the radio signal travel through the 'ether'? How long do the acoustic waves need to travel from the loudspeaker to the ear of the listener? How long does it take for someone who is at the 'live' performance and sits 34 m from the loudspeakers to receive the acoustic waves?
9. What would actually happen with the length of an object, which moves with double the speed of light? And how slowly would such a fast-moving clock tick??
10. Yet another Pythagorean Triple: The root is also quite pretty if  $v/c$  takes the values  $5/13$  or  $12/13$  ... The pair  $3/5$  and  $4/5$  should already be familiar to you.
11. Again two rockets, flying past each other at high speed: A measures when flying by that the two rockets are the same length. What does B measure?  
a) A is the same length as B      b) A is longer than B      c) A is shorter than B ?
12. Are fast-moving clocks, which are synchronized in their system, **really** synchronized or not? The question is just as meaningful as asking about the season: Is it now **really** winter or summer? If you are not sure, then call your uncle in Australia...
13. (Challenging) Derive the length contraction from observing a fast-moving light clock that is lying on its side! The flash of light travels back and forth in the same direction as the clock is moving and thus the 'tick' and 'tock' are not equivalently long to an observer at rest...

We have now described the relativity of objective time measurement. That subjective time experience is 'malleable' is well-known. Salvador Dali's 'The Persistence of Memory' fits both perspectives quite well:



Einstein once illustrated the subjectivity of time experience in the following way:

“An hour sitting with a pretty girl on a park bench passes like a minute, but a minute sitting on a hot stove seems like an hour.” [17-247]